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Leptonic dipole moments in the left-right supersymmetric model

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Abstract. The observation of neutrino oscillations imposes a pattern of mixing in both the sneutrino and charged slepton sectors. On the other hand, the apparent 2.6σ deviation of the anomalous magnetic moment of the muon from the standard model value favors a scenario beyond the standard model. We show that, in a supersymmetric model with left-right symmetry, which provides an explanation for both phenomena, the relationship between flavor conserving dipole moments, such as the magnetic and the electric dipole moments, and flavor violating dipole moments, such as $\mu \to e\gamma$, $\tau \to \mu\gamma$ and $\tau \to e\gamma$, is quite different from that in the MSSM. From general analytic considerations, we derive bounds on the fractional sneutrino mass splittings $\delta^2_{\tilde{\nu}_e\tilde{\nu}_\mu}/m^2_{\tilde{\nu}} \leq 1.5\times10^{-5}$, and the fractional charged slepton splittings $\delta^2_{\tilde{e}\tilde{\mu}}/m^2_{\tilde{l}} \leq 2\times10^{-2}$. For $\tilde{\mu}-\tilde{\tau}$, the mixing is allowed to be maximal. We also comment on the magnitudes and correlations between CP-violating angles coming from electric dipole moments. We supplement the analytical considerations by detailed numerical calculations.

1 Introduction

The recent measurement of the anomalous magnetic moment of the muon at BNL [1] shows a deviation from the result expected in the Standard Model and suggests that a contribution from physics beyond the Standard Model might be necessary to explain the discrepancy. This observation follows recent observations of neutrino oscillations, for both atmospheric and solar neutrinos [2]. Neutrino oscillations are unaccounted for in the Standard Model where neutrinos are exactly massless and therefore cannot mix. The observation of neutrino oscillations also provides the first clear indication of lepton flavor violation.

The most persuasive explanation for the observation of the deviation of the anomalous magnetic moment seems to come from supersymmetry, which provides significant additional contributions to the anomalous magnetic moment operators [3]. The most elegant explanation for the second phenomenon lies in the see-saw mechanism which provides a small mass for the left-handed neutrinos while introducing massive right-handed neutrinos [4]. An explanation for both would require (at least) extending the Standard Model to its supersymmetric version, the Minimal Supersymmetric Standard Model (MSSM) with the addition of right-handed neutrinos. This model has the advantage that it is minimal: however, as a symmetry is seems at best *ad hoc*. It is hoped that extended gauge structures, introduced to provide an elegant framework for the unification of forces [5], could connect the standard model with more fundamental structures such as superstrings, while at the same time resolve the puzzles of the electroweak theory. The Left-Right Supersymmetric (LRSUSY) is perhaps the most natural extension of the minimal supersymmetric model [6–9]. Left-right supersymmetry is based on the group $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, which would then break spontaneously to $SU(2)_L \times U(1)_Y$ [6]. LRSUSY was originally seen as a natural way to suppress rapid proton decay and as a mechanism for providing small neutrino masses [8]. Besides being a plausible symmetry itself, LRSUSY models have the added attractive features that they can be embedded in a supersymmetric grand unified theory such as $SO(10)$ [10]. Another support for left-right theories is provided by building realistic brane worlds from Type I strings. This involves left-right supersymmetry, with supersymmetry broken either at the string scale $M_{SUSY} \approx 10^{10-12}$ GeV, or at $M_{SUSY} \approx 1$ TeV, the difference having implications for gauge unification [11].

The left-right supersymmetric model has interesting implications for lepton flavor violating decays (LFV). High precision measurements of these processes can often provide useful probes of the underlying symmetry of the model. It was recently pointed out that in supersymmetric theories the electromagnetic dipole operators have a very similar structure [13]. This similarity allows the muon anomalous magnetic moment to be related to the electron EDM in terms of the (CP-phase) of the dipole operator and to the rate of lepton flavor violating decays $\mu \to e\gamma$, $\tau \to e\gamma$ and $\tau \to \mu\gamma$ though violations of slepton/sneutrino universality. Such relationships have been investigated in the context of MSSM [13,14].

In this work we show that the relationships found in MSSM are not universal and that quite different relationships can be expected in other models. We do so by in-

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vestigating the dipole moments in the left-right supersymmetric model. This model has some of the features of an supersymmetric $SO(10)$, except that lepton-quark universality is not imposed by unification. Some of the features of the Higgs sector (the choice favored by the see-saw mechanism), as well as left and right slepton mixing, have an effect on the relationships between different processes. Various supersymmetric models already predict lepton flavor violation decays close to experimental limits. Since it is expected that in the near future new searches for LFV processes will be undertaken, with increased event sensitivity, it is important to investigate the predictions for such events in models beyond MSSM. If non-conservation of lepton flavor will be observed by the next generation of experiments, concrete predictions for different models would help disentangle the underlying new symmetry structure.

This paper is organized as follows: we review the LRSUSY model and its sources of flavor violation in Sect. 2. In Sect. 3 we list the dominant contributions to the anomalous magnetic moment, the lepton flavor violating decays $l_i \to l_j\gamma$, and the electron EDM in the presence of slepton and sneutrino mixing. We present relationships between these processes based on analytical considerations in Sect. 4. Our numerical analysis is included in Sect. 5, and we conclude in Sect. 6.

2 The left-right supersymmetric model and lepton flavor violation

The LRSUSY symmetry group, $SU(2)_L \times SU(2)_R \times U(1)_{B-L}$, has matter doublets for both left- and right-handed fermions and their corresponding left- and right-handed scalar partners (sleptons and squarks) [8]. In the gauge sector, corresponding to $SU(2)_L$ and $SU(2)_R$, there are triplet gauge bosons $(W^{+,-}, W^0)_L$, $(W^{+,-}, W^0)_R$ and a singlet gauge boson V corresponding to $U(1)_{B-L}$, together with their superpartners. The Higgs sector of this model consists of two Higgs bi-doublets, $\Phi_u(\frac{1}{2}, \frac{1}{2}, 0)$ and $\Phi_d(\frac{1}{2}, \frac{1}{2}, 0)$, which are required to give masses to the up and down quarks. The spontaneous symmetry breaking of the group $SU(2)_R \times U(1)_{B-L}$ to the hypercharge symmetry group $U(1)_Y$ is accomplished by the vacuum expectation values of a pair of Higgs triplet fields $\Delta_L(1, 0, 2)$ and $\Delta_R(0, 1, 2)$, which transform as the adjoint representation of $SU(2)_R$. The choice of the triplets (versus four doublets) is preferred because with this choice a large Majorana mass can be generated (through the see-saw mechanism) for the right-handed neutrino and a small one for the left-handed neutrino [7]. In addition to the triplets $\Delta_{L,R}$, the model must contain two additional triplets, $\delta_L(1, 0, -2)$ and $\delta_R(0, 1, -2)$, with quantum number $B-L = -2$, to insure cancellation of the anomalies which would otherwise occur in the fermionic sector. The superpotential for the LRSUSY is:

$$\begin{aligned} W = {} & \mathbf{h}_q^{(i)} Q^T \tau_2 \Phi_i \tau_2 Q^c + \mathbf{h}_l^{(i)} L^T \tau_2 \Phi_i \tau_2 L^c \\ & + i(\mathbf{h}_{LR} L^T \tau_2 \Delta_L L + \mathbf{h}_{LR} L^{cT} \tau_2 \Delta_R L^c) \\ & + M_{LR} \left[Tr(\Delta_L \delta_L + \Delta_R \delta_R) \right] + \mu_{ij} Tr(\tau_2 \Phi_i^T \tau_2 \Phi_j) \\ & + W_{NR} \end{aligned} \quad (1)$$

where W_{NR} denotes (possible) non-renormalizable terms arising from higher scale physics or Planck scale effects [15]. The presence of these terms insures that, when the SUSY breaking scale is above M_{W_R}, the ground state is R-parity conserving [16].

The neutral Higgs fields acquire non-zero vacuum expectation values ($VEV's$) through spontaneous symmetry breaking:

$$\langle \Delta \rangle_{L,R} = \begin{pmatrix} 0 & 0 \\ v_{L,R} & 0 \end{pmatrix}, \text{ and } \langle \Phi \rangle_{\mathrm{u,d}} = \begin{pmatrix} \kappa_{u,d} & 0 \\ 0 & \kappa'_{u,d} e^{i\omega} \end{pmatrix}.$$

$\langle \Phi \rangle$ causes the mixing of W_L and W_R bosons with CP-violating phase ω. The Higgs fields acquire non-zero $VEV's$ to break both parity and $SU(2)_R$. In the first stage of breaking, the right-handed gauge bosons, W_R and Z_R acquire masses proportional to v_R and become much heavier than the usual (left-handed) neutral gauge bosons W_L and Z_L, which pick up masses proportional to κ_u and κ_d at the second stage of breaking.

In the supersymmetric sector of the model there are six singly-charged charginos, corresponding to $\tilde{\lambda}_L$, $\tilde{\lambda}_R$, $\tilde{\phi}_u$, $\tilde{\phi}_d$, $\tilde{\Delta}_L^-$, and $\tilde{\Delta}_R^-$. The model also has eleven neutralinos, corresponding to $\tilde{\lambda}_Z$, $\tilde{\lambda}_{Z'}$, $\tilde{\lambda}_V$, $\tilde{\phi}_{1u}^0$,$\tilde{\phi}_{2u}^0$, $\tilde{\phi}_{1d}^0$, $\tilde{\phi}_{2d}^0$, $\tilde{\Delta}_L^0$, $\tilde{\Delta}_R^0$ $\tilde{\delta}_L^0$, and $\tilde{\delta}_R^0$. It has been shown that, in the scalar sector, the left-triplet Δ_L couplings can be neglected in phenomenological analyses of muon and tau decays [17]. Although Δ_L is not necessary for symmetry breaking [9], and is introduced only for preserving left-right symmetry, both Δ_L^{--} and its right-handed counterpart Δ_R^{--} play very important roles in phenomenological studies of the LRSUSY model. We include them both in our formal expressions, but only the $\tilde{\Delta}_R$ contribution in the numerical analysis.

The sources of flavor violation in the LRSUSY model come from either the Yukawa potential or the trilinear scalar coupling.

The interaction of fermions with scalar (Higgs) fields has the following form:

$$\begin{aligned} \mathcal{L}_Y &= \mathbf{h}_u \bar{Q}_L \Phi_u Q_R + \mathbf{h}_d \bar{Q}_L \Phi_d Q_R + \mathbf{h}_\nu \bar{L}_L \Phi_u L_R \\ & \quad + \mathbf{h}_e \bar{L}_L \Phi_d L_R + H.c.; \\ \mathcal{L}_M &= i\mathbf{h}_{LR}(L_L^T C^{-1} \tau_2 \Delta_L L_L + L_R^T C^{-1} \tau_2 \Delta_R L_R) \\ & \quad + H.c. \end{aligned} \quad (2)$$

where $\mathbf{h}_u$, $\mathbf{h}_d$, $\mathbf{h}_\nu$ and $\mathbf{h}_e$ are the Yukawa couplings for the up and down quarks and neutrino and electron, respectively, and $\mathbf{h}_{LR}$ is the coupling for the triplet Higgs bosons. LR symmetry requires all $\mathbf{h}$-matrices to be Hermitean in the generation space and $\mathbf{h}_{LR}$ matrix to be symmetric. The Yukawa matrices represent misalignment between the particle and sparticle bases in flavor space and thus cause flavor violation. In addition soft supersymmetry breaking terms which generate masses for the charged slepton fields also induce LFV. The SUSY-breaking terms for the Higgs bosons and lepton sector in LRSUSY is given by:

$$-\mathcal{L}_{soft} = -\Big[\mathbf{A}_l^i \mathbf{h}_l^{(i)} \tilde{L}^T \tau_2 \Phi_i \tau_2 \tilde{L}^c + i\mathbf{A}_{LR} \mathbf{h}_{LR}$$

$$(m_{\tilde{l}}^2)_{eff} = \begin{pmatrix} \tilde{m}_L^2 + D_L & (\tilde{m}_L)_{21}^2 & (\tilde{m}_L)_{31} & \mathcal{A}_e & 0 & 0 \\ (\tilde{m}_L)_{21}^2 & \tilde{m}_L^2 + D_L & (\tilde{m}_L)_{32}^2 & 0 & \mathcal{A}_\mu & 0 \\ (\tilde{m}_L)_{31}^2 & (\tilde{m}_L)_{32}^2 & \tilde{m}_L^2 + D_L & 0 & 0 & \mathcal{A}_\tau \\ \mathcal{A}_e & 0 & 0 & \tilde{m}_R^2 + D_R & (\tilde{m}_R)_{21}^2 & (\tilde{m}_R)_{31}^2 \\ 0 & \mathcal{A}_\mu & 0 & (\tilde{m}_R)_{21}^2 & \tilde{m}_R^2 + D_R & (\tilde{m}_R)_{32}^2 \\ 0 & 0 & \mathcal{A}_\tau & (\tilde{m}_R)_{31}^2 & (\tilde{m}_R)_{32}^2 & \tilde{m}_R^2 + D_R \end{pmatrix}, \quad (4)$$

$$(m_{\tilde{\nu}}^2)_{eff} = \begin{pmatrix} m_L^2 - \mathcal{A}'_\nu (A_\nu - 2A_N)(m_D M^{-2} m_D^\dagger) & \mathcal{A}'^*_\nu (m_D M^{-1} m_D^\dagger) \\ \mathcal{A}'_\nu (m_D M^{-1} m_D^\dagger)^* & m_L^2 - \mathcal{A}'_\nu (A_\nu - 2A_N)(m_D M^{-2} m_D^\dagger) \end{pmatrix} \quad (6)$$

$$\times (\tilde{L}^T \tau_2 \Delta_L \tilde{L} + L^{cT} \tau_2 \Delta_R \tilde{L}^c) + m_\Phi^{(ij)2} \Phi_i^\dagger \Phi_j \Big]$$
$$+ \Big[(m_L^2)_{ij} \tilde{l}_{Li}^\dagger \tilde{l}_{Lj} + (m_R^2)_{ij} \tilde{l}_{Ri}^\dagger \tilde{l}_{Rj} + (M_N^2)_{ij} \tilde{N}_i^* \tilde{N}_j^* \Big]$$
$$+ M_{LR}^2 \left[Tr(\Delta_R \delta_R) + Tr(\Delta_L \delta_L) \right]$$
$$+ B\mu_{ij} \Phi_i \Phi_j \quad (3)$$

where the **A**-matrices (A_u, A_d, A_ν and A_e) are of similar form to the Yukawa couplings and provide additional sources of flavor violation, B is a mass term and $\tilde{N}$ is the scalar component of the right-handed neutrino supermultiplet. The inter-generational slepton mixing ($\tilde{e}$, $\tilde{\mu}$ and $\tilde{\tau}$) and also left-right slepton mixing ($\tilde{e}_L, \tilde{e}_R, \tilde{\mu}_L, \tilde{\mu}_R, \tilde{\tau}_L, \tilde{\tau}_R$) cause the off-diagonal nature of the matrices, and therefore are responsible for flavor violation. The slepton mass matrix, which arises as a result of the renormalization group evolution from the Λ_{GUT} scale, is, incorporating some elements of the left-right symmetry [18]: (see (4) on top of the page) where $\mathcal{A}_l = A_l m_l + m_l \mu \tan\beta$, $(l = e, \mu, \tau)$, $D_L = M_{Z_L}^2 (T_3/2 + \sin^2\theta_W)\cos 2\beta + M_{Z_R}^2 \sin^2\theta_W / \sin 2\theta_W \cos 2\beta$, and $D_R = -M_{Z_L}^2 \sin^2\theta_W \cos 2\beta + M_{Z_R}^2 (T_3/2 - \sin^2\theta_W / \cos 2\theta_W)\sin 2\beta$. We denote the charged slepton mixing matrix by $V^{L,R}$ and express the slepton mixing as:

$$\tilde{l}_{\alpha L,R} = V_{\alpha i}^{L,R} \tilde{l}_i^{L,R} \quad (5)$$

with $\alpha = e, \mu, \tau$ and $i = 1, 2, 3$. This approximates the left-left and right-right slepton mixings by block-diagonal matrices, while allowing the left-right mixings to be proportional to the values of the trilinear parameters $\mathcal{A}_l$.

The full mass for left- and right-handed sneutrino has, in general, a complicated 12×12 matrix structure, but is possible to construct an effective 6×6 matrix for the light sneutrinos using the see-saw mechanism [19]: (see (6) on top of the page) where $\mathcal{A}'_\nu \sim 2A_\nu + A_N + 2\mu \cot\beta$. Note that in LRSUSY model, the left-handed neutrino mass is allowed to be nonzero, but can be made small through the see-saw mechanism, as long as the right-handed neutrino is very heavy (masses of order 10^{10} TeV or so are consistent with the 1 eV limit on the left-handed neutrino mass). Despite the presence of the two scalar neutrinos, the mixing between the right-handed and the left-handed sneutrinos is small, due to the see-saw mechanism in the sfermion sector. The left-right elements of the sneutrino mass matrix are proportional to the Dirac neutrino mass, which can be significant. But the right-right element of the sneutrino mass matrix is very heavy, so the mixing of sneutrino will be supressed by the inverse M_N^2, the right-handed neutrino mass. As opposed to the charged slepton sector, in the light sneutrino sector the Dirac terms do not induce considerable mixing [20]. We expect the off-diagonal terms in the sneutrino mass matrix to mix almost degenerate states and thus to affect flavor violating decays less than the charged slepton mixing. We denote the light sneutrino mixing matrix by K and express their mixing as:

$$\tilde{\nu}_\alpha = K_{\alpha i} \tilde{\nu}_i \quad (7)$$

with $\alpha = \nu_e, \nu_\mu, \nu_\tau$ and $i = 1, 2, 3$.

3 Dominant contributions to leptonic dipole moments

All of the high precision measurements which involve the coupling of a photon to Standard Model fermions are derived from effective operators of the electromagnetic dipole form. The magnitude of these operators depends on the details of the model: heavy particle spectrum, scale of symmetry breaking, interactions which violate the symmetries. The dipole operator responsible for the anomalous magnetic moment of the muon has the form:

$$\mathcal{M}_\mu = \frac{ie}{2m_\mu} \bar{u}_\mu(p_2)(a_{L\mu} P_L + a_{R\mu} P_R)\sigma^{\mu\nu} q_\nu u_\mu(p_1) A_\mu \quad (8)$$

The lepton-flavor violating dipole operator for the process $\mu \to e\gamma$ has the form:

$$\mathcal{M}_{\mu e\gamma} = \frac{ie}{2m_\mu} \bar{u}_e(p_2)\sigma^{\mu\nu} q_\nu (a_{L\mu e\gamma} P_L + a_{R\mu e\gamma} P_R) \times u_\mu(p_1) A_\mu + h.c., \quad (9)$$

and the dipole operator responsible for the electron electric dipole moment is:

$$\mathcal{D}_e = -\frac{i}{2} d_e \bar{u}_e(p_2)\sigma_{\mu\nu} q_\mu \gamma_5 u_e(p_1) \quad (10)$$

Next, we give their main contributions in LRSUSY.

3.1 $(g-2)_\mu$

The new measurement for the muon anomalous magnetic moment a_μ [1]:

$$a_\mu^{exp} - a_\mu^{SM} = (4.26 \pm 1.65) \times 10^{-9} \quad (11)$$

If the deviation can be attributed to new physics effects, then at 90% CL δa_μ^{NP} must lie in the range:

$$2.15 \times 10^{-9} \le \delta a_\mu^{NP} \le 6.37 \times 10^{-9} \quad (12)$$

The contribution to the anomalous magnetic moment of the muon has been analysed in [8] in the absence of sneutrino and slepton mixing. However, since some contributions to the magnetic moment were omitted, such as the ones coming from the doubly-charged higgsinos or explicitly from the left-right higgsino, we give below the dominant contributions, and for completness, include all the contributions (with slepton/sneutrino mixing) in Appendix B. The contributions of LRSUSY model to the anomalous magnetic moment of the muon fall into three categories: chargino-sneutrino, neutralino-charged-slepton and doubly charged higgsinos-charged sleptons. Since the dipole moment involves a chirality flip, the graphs can be classified according to where the chirality flip occurs: on the external lepton leg, on the internal slepton leg, and at the vertex. Because chirality flipping results in proportionality to the mass of the fermion where chirality is flipped, the dominant contributions come from flipping chirality internally. (In those cases the loop function is also largest). These dominant contributions are:

$$a_\mu = a_\mu^c + a_\mu^n + a_\mu^\Phi \quad (13)$$

and are given in detail below. The chargino-neutrino contribution (with vertex chirality flip, Fig. 1f) is:

$$a_{L\mu}^c = \frac{m_\mu}{16\pi^2} M_{\chi^\pm} g Y_\mu Re[U_{k1}^{-*} U_{k1}^{+*}] \times \left[|K_{\mu 1}|^2 \frac{h(x_{ke})}{m_{\tilde\nu_e}^2} + |K_{\mu 2}|^2 \frac{h(x_{k\mu})}{m_{\tilde\nu_\mu}^2} + |K_{\mu 3}|^2 \frac{h(x_{k\tau})}{m_{\tilde\nu_\tau}^2}\right] \quad (14)$$

The neutralino-left slepton contribution (with vertex chirality flip, Fig. 1g):

$$a_{L\mu}^{n,1} = \frac{m_\mu}{16\pi^2} M_{\chi^0} \sqrt{2} g Y_\mu \left(N_{W_L k}^0 + \tan\theta_W^2 N_{Bk}^0\right) N_{H_k}^0 \times \left[|V_{\mu 1}^L|^2 2\frac{j(y_{ke_L})}{m_{\tilde e_L}^2} + |V_{\mu 2}^L|^2 \frac{j(y_{k\mu_L})}{m_{\tilde\mu_L}^2} + |V_{\mu 3}^L|^2 \frac{j(y_{k\tau_L})}{m_{\tilde\tau_L}^2}\right] \quad (15)$$

and the neutralino-right slepton contribution is (with vertex chirality flip, Fig. 1h):

$$a_{R\mu}^{n,1} = \frac{m_\mu}{16\sqrt{2}\pi^2} M_{\chi^0} g Y_\mu \left(N_{W_R k}^0 - 2\tan\theta_W^2 N_{Bk}^0\right) N_{H_k}^0 \times \left[|V_{\mu 1}^R|^2 \frac{j(y_{ke_R})}{m_{\tilde e_R}^2} + |V_{\mu 2}^R|^2 \frac{j(y_{k\mu_R})}{m_{\tilde\mu_R}^2} + |V_{\mu 3}^R|^2 \frac{j(y_{k\tau_R})}{m_{\tilde\tau_R}^2}\right] \quad (16)$$

The neutralino-left slepton contribution (with internal line chirality flip, Fig. 1i) is:

$$a_{L\mu}^{n,2} = \frac{m_\mu}{16\pi^2} M_{\chi^0} g^2 \left(N_{W_R k}^0 - 2\tan\theta_W N_{Bk}^0\right) \times \left(N_{W_L k}^0 + \tan\theta_W^2 N_{Bk}^0\right) \times \left[\frac{\mathcal{A}_e}{m_{\tilde e_R}^2} V_{\mu 1}^L V_{\mu 1}^{R*} \frac{j(y_{ke_L})}{m_{\tilde e_L}^2} + \frac{\mathcal{A}_\mu}{m_{\tilde\mu_R}^2} V_{\mu 2}^L V_{\mu 2}^{R*} \frac{j(y_{k\mu_L})}{m_{\tilde\mu_L}^2} + \frac{\mathcal{A}_\tau}{m_{\tilde\tau_R}^2} V_{\mu 3}^L V_{\mu 3}^{R*} \frac{j(y_{k\tau_L})}{m_{\tilde\tau_L}^2}\right] \quad (17)$$

and the neutralino-right slepton contribution is (with internal line chirality flip, Fig. 1j):

$$a_{R\mu}^{n,2} = \frac{m_\mu}{16\pi^2} M_{\chi^0} g^2 \left(N_{W_L k}^0 + \tan\theta_W^2 N_{Bk}^0\right) \times \left(N_{W_R k}^0 - 2\tan\theta_W N_{Bk}^0\right) \times \left[\frac{\mathcal{A}_e}{m_{\tilde e_L}^2} V_{\mu 1}^R V_{\mu 1}^{L*} \frac{j(y_{ke_R})}{m_{\tilde e_R}^2} + \frac{\mathcal{A}_\mu}{m_{\tilde\mu_L}^2} V_{\mu 2}^R V_{\mu 2}^{L*} \frac{j(y_{k\mu_R})}{m_{\tilde\mu_R}^2} + \frac{\mathcal{A}_\tau}{m_{\tilde e_L}^2} V_{\mu 3}^R V_{\mu 3}^{L*} \frac{j(y_{k\tau_R})}{m_{\tilde\tau_R}^2}\right] \quad (18)$$

In addition to these standard $(g-2)_\mu$ contributions, there exists a potentially large LRSUSY contribution, proportional to m_τ, from the higgsino partner of the FCNC Higgs Φ_{2u}^0, which couples to the left-right fermion-sfermion vertex [21]. The left-right Higgsino $\tilde\Phi_{2u}^0$-slepton contribution is, Fig. 1k:

$$a_\mu^\phi = \frac{2Y_{\mu\tau}^2}{(4\pi)^2} m_\mu m_\tau Re[\mu\tan\beta - A_\tau^*] \frac{M_{\tilde\Phi_{2u}^0}}{m_{\tilde\tau_R}^2 - m_{\tilde\tau_L}^2} \times \left[\frac{j(y_{\tau R})}{m_{\tilde\tau_R}^2} - \frac{j(y_{\tau_L})}{m_{\tilde\tau_L}^2}\right] \quad (19)$$

where $Y_{\mu\tau} = Y_{\nu_3} V_{\tau 3} V_{\tau 2}^*$, with $V_{\alpha j}$ the charged slepton mixing matrices. This contribution is dominant for large Y_{ν_3}, unless the field $\tilde\phi_{2u}^0$ is heavy and decouples from the low-energy spectrum at v_R.

The arguments of the loop functions (defined in Appendix B) are: $x_{k\alpha} = M_k^2/m_{\tilde\nu_\alpha}^2$ where k represents the chargino, and α represents the sneutrino; and $y_{k\alpha} = M_k^2/m_{\tilde l_\alpha}^2$ where k represents the neutralino, and α represents the slepton.

3.2 $\mu \to e\gamma$

The amplitude of the $\mu \to e\gamma$ transition, written in the form of the usual dipole-type interaction, leads to the branching ratio:

$$\Gamma_{\mu\to e\gamma} = \frac{1}{16\pi^2}(|a_{L\mu e\gamma}|^2 + |a_{R\mu e\gamma}|^2) m_\mu^5 \quad (20)$$

Comparing it with the standard decay width, $\Gamma_\mu = \frac{1}{192\pi^3} G_F^2 m_\mu^5$ and using the recent experimental constraint on the branching ratio [22]:

$$B.R.(\mu \to e\gamma) \le 1.2 \times 10^{-11} \quad (21)$$

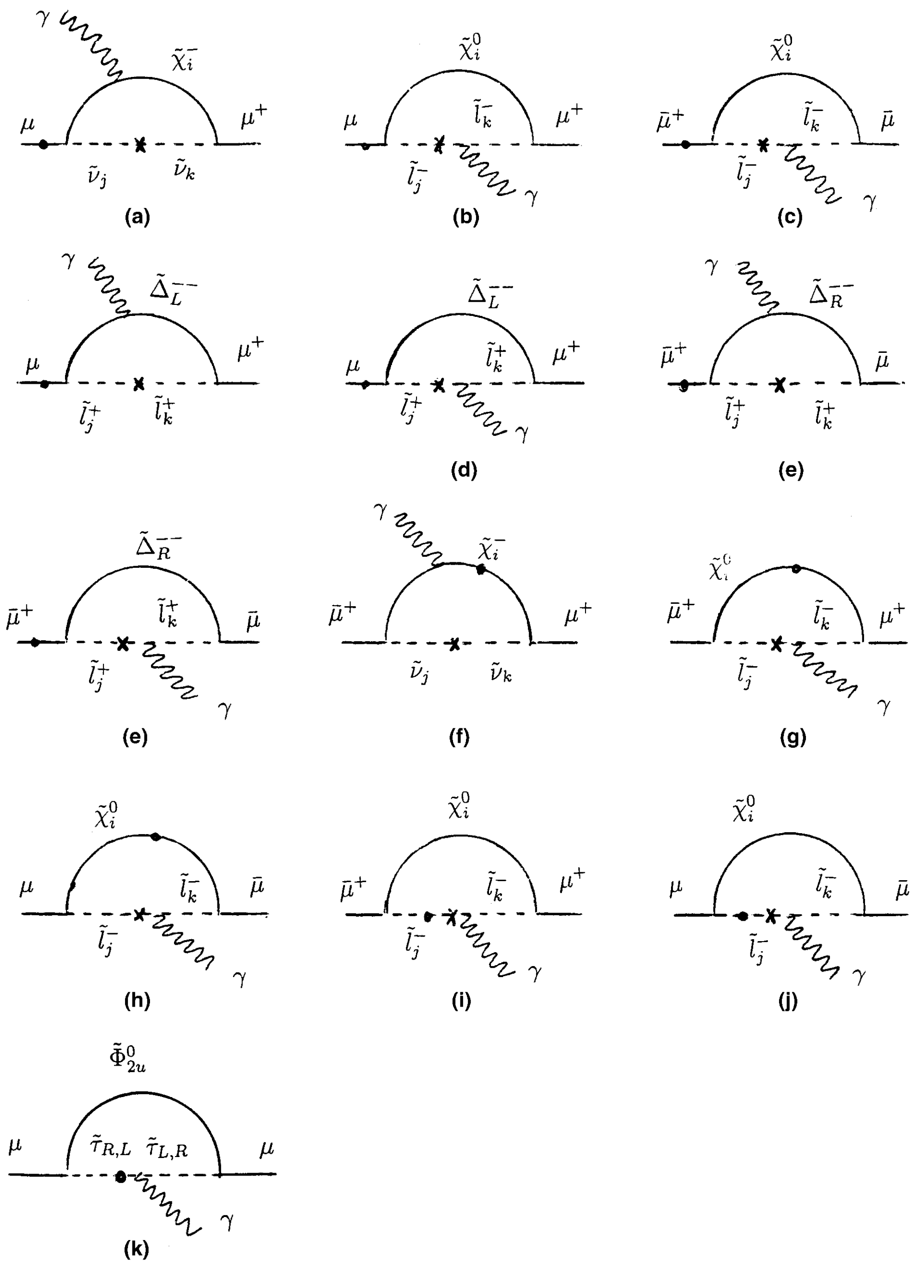

Fig. 1a–k. One-loop contributions to the anomalous magnetic moment of the muon: **a** charginos, left-handed fermions, with an external chirality flip; **b** neutralinos, left-handed fermions, with an external chirality flip; **c** neutralinos, right-handed fermions, with an external chirality flip; **d** doubly-charged Higgsinos, left-handed fermions, with an external chirality flip; **e** doubly-charged Higgsinos, right-handed fermions, with an external chirality flip; **f** charginos with a vertex chirality flip; **g** neutralinos, left-handed sleptons with a vertex chirality flip; **h** neutralinos, right-handed fermions, with a vertex chirality flip; **i** neutralinos, left-handed fermions, with an internal line chirality flip; **j** neutralinos, right-handed fermions, with an internal line chirality flip; and **k** left-right higgsino with internal line chirality flip. Here $\tilde{\chi}_i^{\pm}$ represents a chargino state and i runs from 1 to 8; $\tilde{\chi}_i^0$ represents a neutralino state and i runs from 1 to 11; $\tilde{\Delta}_{L,R}^{--}$ represents the doubly-charged Higgsino state; and $\tilde{\Phi}_u^0$ is the flavor-changing left-right higgsino; $\tilde{l}$, $\tilde{\nu}$ represent slepton and left-handed sneutrino, respectively. The crosses represent slepton or sneutrino flavor mixing and the dots chirality flips

we get the following limit on the dipole amplitude:

$$|d| = \sqrt{(|a_{L\mu e\gamma}|^2 + |a_{R\mu e\gamma}|^2)/2} < 1.73 \cdot 10^{-26}\ e \cdot cm \quad (22)$$

The contributions to the lepton flavor violation dipole are related to the muon $(g-2)_\mu$ graphs by a simple relacement of the outgoing muon with an electron. These contributions can be written in pairs, with the same particles in the loop but the incoming and outgoing muons having different chiralities, leading to distinct contributions to the amplitude for a_L and a_R. The dominant contributions are:

$$a_{\mu e\gamma} = a^c_{\mu e\gamma} + a^n_{\mu e\gamma} + a^{\Phi}_{\mu e\gamma} \quad (23)$$

and are given in detail below. The chargino-neutrino contribution (with vertex chirality flip) is:

$$a^c_{L\mu e\gamma} = \frac{g^2 e}{32\pi^2}\frac{M_{\chi^\pm}}{\sqrt{2}M_W\cos\beta}Re[U^{-*}_{k1}U^{+*}_{k1}]\left[K_{\mu 1}K^*_{e1}\frac{h(x_{ke})}{m^2_{\tilde{\nu}_e}} + K_{\mu 2}K^*_{e2}\frac{h(x_{k\mu})}{m^2_{\tilde{\nu}_\mu}} + K_{\mu 3}K^*_{e3}\frac{h(x_{k\tau})}{m^2_{\tilde{\nu}_\tau}}\right] \quad (24)$$

The neutralino-left slepton contribution (with vertex chirality flip) is:

$$a^{n,1}_{L\mu e\gamma} = \frac{g^2 e}{32\pi^2}\frac{M_{\chi^0}}{2M_W\cos\beta}\left(N^0_{W_L k} + \tan\theta^2_W N^0_{Bk}\right)N^0_{H_k} \times \left[V^L_{\mu 1}V^{L*}_{e1}\frac{j(y_{ke_L})}{m^2_{\tilde{e}_L}} + V^L_{\mu 2}V^{L*}_{e2}\frac{j(y_{k\mu_L})}{m^2_{\tilde{\mu}_L}}\right.$$

$$\times +V^L_{\mu 3}V^{L*}_{e3}\frac{j(y_{k\tau_L})}{m^2_{\tilde{\tau}_L}}\Bigg] \quad (25)$$

The neutralino-right slepton contribution (with vertex chirality flip) is:

$$a^{n,1}_{R\mu e\gamma} = \frac{g^2 e}{32\pi^2}\frac{M_{\chi^0}}{4M_W\cos\beta}\left(N^0_{W_Lk} - 2\tan\theta^2_W N^0_{Bk}\right)N^0_{H_k} \times\left[V^R_{\mu 1}V^{R*}_{e1}\frac{j(y_{ke_R})}{m^2_{\tilde{e}_R}} + V^R_{\mu 2}V^{R*}_{e2}\frac{j(y_{k\mu_R})}{m^2_{\tilde{\mu}_R}} + V^R_{\mu 3}V^{R*}_{e3}\frac{j(y_{k\tau_R})}{m^2_{\tilde{\tau}_R}}\right] \quad (26)$$

The neutralino-left slepton contribution (with internal line chirality flip) is:

$$a^{n,2}_{L\mu e\gamma} = \frac{g^2 e}{32\pi^2}M_{\chi^0}\left(N^0_{W_Rk} - 2\tan\theta_W N^0_{Bk}\right) \quad (27)$$
$$\times\left(N^0_{W_Lk} + \tan\theta^2_W N^0_{Bk}\right)\left[\frac{\mathcal{A}_e}{m^2_{\tilde{e}_R}}V^L_{\mu 1}V^{R*}_{e1}\frac{j(y_{ke_L})}{m^2_{\tilde{e}_L}} + \frac{\mathcal{A}_\mu}{m^2_{\tilde{\mu}_R}}V^L_{\mu 2}V^{R*}_{e2}\frac{j(y_{k\mu_L})}{m^2_{\tilde{\mu}_L}} + \frac{\mathcal{A}_\tau}{m^2_{\tilde{\tau}_R}}V^L_{\mu 3}V^{R*}_{e3}\frac{j(y_{k\tau_L})}{m^2_{\tilde{\tau}_L}}\right]$$

and the neutralino-right slepton contribution is (with internal line chirality flip):

$$a^{n,2}_{R\mu e\gamma} = \frac{g^2 e}{32\pi^2}M_{\chi^0}\left(N^0_{W_Lk} + \tan\theta^2_W N^0_{Bk}\right) \quad (28)$$
$$\times\left(N^0_{W_Rk} - 2\tan\theta_W N^0_{Bk}\right)\left[\frac{\mathcal{A}_e}{m^2_{\tilde{e}_L}}V^R_{\mu 1}V^{L*}_{e1}\frac{j(y_{ke_R})}{m^2_{\tilde{e}_R}} + \frac{\mathcal{A}_\mu}{m^2_{\tilde{\mu}_L}}V^R_{\mu 2}V^{L*}_{e2}\frac{j(y_{k\mu_R})}{m^2_{\tilde{\mu}_R}} + \frac{\mathcal{A}_\tau}{m^2_{\tilde{\tau}_L}}V^R_{\mu 3}V^{L*}_{e3}\frac{j(y_{k\tau_R})}{m^2_{\tilde{\tau}_R}}\right]$$

Finally the corresponding left-right Higgsino-slepton contribution is:

$$a^{\phi}_{\mu e\gamma} = \frac{eY_{\mu\tau}Y^*_{\tau e}}{(4\pi)^2}m_\tau Re[\mu\tan\beta - A^*_\tau]\frac{M_{\tilde{\Phi}^0_{2u}}}{m^2_{\tilde{\tau}_R} - m^2_{\tilde{\tau}_L}} \times\left[\frac{j(y_{\tau R})}{m^2_{\tilde{\tau}_R}} - \frac{j(y_{\tau_L})}{m^2_{\tilde{\tau}_L}}\right] \quad (29)$$

where $Y_{e\tau} = Y_{\nu_3}V_{\tau 3}V^*_{\tau 1}$, and $V_{\alpha j}$ the charged slepton mixing matrices.

3.3 The electric dipole moment of the electron

The contributions to the electric dipole moment of the electron come from the same graphs as the muon anomalous magnetic moment, except that both the incoming and outgoing muon have to be replaced by electrons. Since a non-vanishing d_f in the SM results in fermion chirality flip, both CP violation and $SU(2)_L$ symmetry breaking are required. The corresponing contribution will depend on the phases in the model, which are not universal, even if the soft-breaking masses are assumed to be universal. There are several CP-violating phases allowed in the left-right supersymmetric model [24]. Some appear in the gaugino masses for the $SU(2)_L$, $SU(2)_R$ and $U(1)_{B-L}$: $M_i = |M_i|exp(i\phi_i)$. Some appear in the soft-symmetry breaking Lagrangian, in the trilinear soft breaking parameter $A_0 = |A_0|exp(i\alpha_0)$ and in the quadratic soft-breaking $B_0 = |B_0|exp(i\theta_{B_0})$. There are also CP-violating phases in the Higgs mixing parameter in the superpotential: $\mu_{ij0} = |\mu_{ij0}|exp(i\theta_{\mu_{ij}})$. As in MSSM, one can always set one of the gaugino phases to zero and we choose $\phi_2 = 0$. Since we would like to compare the results obtained in LRSUSY with the ones obtained in MSSM, we chose a minimal set of non-zero phases to coincide to mSUGRA [14]. We eliminate all phases in favor of two which we choose to be $\theta_{ij\mu} \equiv \delta_{ij}\theta_\mu$ and ϕ_1. The dominant contributions to the electron dipole moment comes then from the chargino-sneutrino and the neutralino-slepton graphs as below:

$$d_e = d^c_e + d^n_e + d^{\Phi}_e \quad (30)$$

and are given in detail below. The chargino-neutrino contribution (with vertex chirality flip) is:

$$\frac{d^c_{L\mu}}{e} = -\frac{m_e}{32\pi^2}M_{\chi^\pm}gY_\mu Re[U^{-*}_{k1}U^{+*}_{k1}]\left[|K_{e1}|^2\frac{h(x_{ke})}{m^2_{\tilde{\nu}_e}} \times +|K_{e2}|^2\frac{h(x_{k\mu})}{m^2_{\tilde{\nu}_\mu}} + |K_{e3}|^2\frac{h(x_{k\tau})}{m^2_{\tilde{\nu}_\tau}}\right]\sin\theta_\mu \quad (31)$$

The neutralino-left slepton contribution (with vertex chirality flip) is:

$$\frac{d^{n,1}_{Le}}{e} = -\frac{m_e}{32\pi^2}M_{\chi^0}\sqrt{2}gY_\mu\left(N^0_{W_Lk} + \tan\theta^2_W N^0_{Bk}\right)N^0_{H_k} \times\left[|V^L_{e1}|^2 2\frac{j(y_{ke_L})}{m^2_{\tilde{e}_L}} + |V^L_{e2}|^2\frac{j(y_{k\mu_L})}{m^2_{\tilde{\mu}_L}} + |V^L_{e3}|^2\frac{j(y_{k\tau_L})}{m^2_{\tilde{\tau}_L}}\right] \times\sin(\theta_\mu + \phi_1) \quad (32)$$

and the neutralino-right slepton contribution is (with vertex chirality flip):

$$\frac{d^{n,1}_{Re}}{e} = -\frac{g^2}{32\pi^2}\left(N^0_{W_Rk} - 2\tan\theta^2_W N^0_{Bk}\right)N^0_{H_k} \times\left[|V^R_{e1}|^2\frac{j(y_{ke_R})}{m^2_{\tilde{e}_R}} + |V^R_{e2}|^2\frac{j(y_{k\mu_R})}{m^2_{\tilde{\mu}_R}} + |V^R_{e3}|^2\frac{j(y_{k\tau_R})}{m^2_{\tilde{\tau}_R}}\right] \times\sin(\theta_\mu + \phi_1) \quad (33)$$

The neutralino-left slepton contribution (with internal line chirality flip) is:

$$\frac{d^{n,2}_{Le}}{e} = -\frac{m_e}{32\pi^2}M_{\chi^0}g^2\left(N^0_{W_Rk} - 2\tan\theta_W N^0_{Bk}\right)$$
$$\times\left(N^0_{W_Lk} + \tan\theta^2_W N^0_{Bk}\right)\left[\frac{\mathcal{A}_e}{m^2_{\tilde{e}_R}}V^L_{e1}V^{R*}_{e1}\frac{j(y_{ke_L})}{m^2_{\tilde{e}_L}} + \frac{\mathcal{A}_\mu}{m^2_{\tilde{\mu}_R}}V^L_{e2}V^{R*}_{e2}\frac{j(y_{k\mu_L})}{m^2_{\tilde{\mu}_L}} + \frac{\mathcal{A}_\tau}{m^2_{\tilde{\tau}_R}}V^L_{e3}V^{R*}_{e3}\frac{j(y_{k\tau_L})}{m^2_{\tilde{\tau}_L}}\right] \times\sin(\theta_\mu + \phi_1) \quad (34)$$

and the neutralino-right slepton contribution is (with internal line chirality flip):

$$\frac{d^{n,2}_{Re}}{e} = -\frac{m_e}{32\pi^2} M_{\chi^0} g^2 \left(N^0_{W_L k} + \tan\theta^2_W N^0_{Bk}\right) \times \left(N^0_{W_R k} - 2\tan\theta_W N^0_{Bk}\right) \left[\frac{\mathcal{A}_e}{m^2_{\tilde{e}_L}} V^R_{e1} V^{L*}_{e1} \frac{j(y_{k e_R})}{m^2_{\tilde{e}_R}} + \frac{\mathcal{A}_\mu}{m^2_{\tilde{\mu}_L}} V^R_{e2} V^{L*}_{e2} \frac{j(y_{k\mu_R})}{m^2_{\tilde{\mu}_R}} + \frac{\mathcal{A}_\tau}{m^2_{\tilde{e}_L}} V^R_{e3} V^{L*}_{e3} \frac{j(y_{k\tau_R})}{m^2_{\tilde{\tau}_R}}\right] \times \sin(\theta_\mu + \phi_1) \quad (35)$$

The left-right Higgsino-slepton contribution is:

$$\frac{d^{\phi}_{Le}}{e} = -\frac{Y^3_\tau Y_\mu J}{(4\pi)^2} m_e m_\tau Re[\mu \tan\beta - A^*_\tau] \frac{M_{\tilde{\Phi}^0_{2u}}}{m^2_{\tilde{\tau}_R} - m^2_{\tilde{\tau}_L}} \times \left[\frac{j(x_R)}{m^2_{\tilde{\tau}_R}} - \frac{j(x_L)}{m^2_{\tilde{\tau}_L}}\right] \quad (36)$$

where $J = Im(V_{\mu 1} V^*_{\tau 1} V_{\tau 2} V^*_{\mu 2})$ is the leptonic CP-odd rephasing invariant.

Experimentally, the EDM of the electron is one of the most restrictive parameters in the Particle Data Booklet, the present experimental upper limit being $d_e \leq 4.3 \cdot 10^{-27}\,\mathrm{e\,cm}$ [25].

4 Relationships between dipole operators: The analytical approach

The electroweak sector of LRSUSY contains three gauge couplings and three gaugino masses. The Higgs sector of the Lagrangian contains the scalar masses m_{Φ_u}, m_{Φ_d}, M_{LR} as well as the parameters μ_{ij} and B. The remaining part of the Lagrangian contains, in the flavor sector, fermion Yukawa matrices for both left and right-handed fermions, four trilinear scalar coupling matrices and scalar mass matrices. A analysis in terms of these parameters would yield some of the general features of the model and will be performed in the next section.

However, some of the features of the model are apparent though some general analytical considerations. Such features of dipole moments have been analysed in the context of the MSSM. We are interested in the general features of the relationships between lepton-flavor conserving and lepton flavor-violating dipole operators in LRSUSY for the purpose of showing that these features are not universal. Mixings between the first and the second generations will be probed at the PSI experiment [26], which is expected to probe $\mu \to e\gamma$ decay rates to 10^{-14} branching ratio. Such high precision experimental data should be able to disentangle these relationships in the future, and therefore provide an insight into the gauge symmetry of the leptonic sector of supersymmetry.

The supersymmetry breaking masses of sleptons are given by the renormalization group equations (RGE). Since the Yukawa couplings and the A terms are flavor-violating, they will induce LFV terms in the off-diagonal mass matrices. The soft mass parameters will evolve such that the slepton masses receive corrections proportional to the Dirac mass.

Independent of which diagrams dominate, up to small corrections proportional to powers of the lepton Yukawa couplings, all contributions to the lepton dipole operators are proportional to a single power of the lepton mass. With slepton universality and proportionality of the scalar trilinear soft terms, this implies that dipole operators for different leptons are related simply by ratios of the Yukawa couplings or equivalently lepton masses. For example, for the electron and muon we expect:

$$a^{(L,R)}_{\mu e\gamma} \simeq -e\frac{m_e}{2m^2_\mu} a_\mu \delta^{(L,R)}_{ij} \quad (37)$$

$$d_e \simeq -e\frac{m_e}{2m^2_\mu} a_\mu \tan\phi \quad (38)$$

with ϕ some CP-violating phase. However, violations of slepton universality and proportionality can significantly modify the relations above. We assume here that all the A terms are approximately proportional and that the scalar masses are approximately universal. We assume that the SUSY breaking parameters associated with the supersymmetric Yukawa couplings or masses are proportional to the Yukawa coupling constants or masses, and are given as:

$$\begin{aligned}(\tilde{m}^2_L)_{ij} = (\tilde{m}^2_R)_{ij} = (\tilde{m}^2_\nu)_{ij} &= \delta_{ij} m^2_0 \\ m^2_{\tilde{\Phi}_1} = m^2_{\tilde{\Phi}_2} &= m^2_0 \\ A^{ij}_\nu = f_{\nu_{ij}} a_0, \quad A^{ij}_l &= f_{l_{ij}} a_0 \\ B^{ij}_\nu = M_{\nu_i\nu_j} b_0, \quad B_\Phi &= \mu b_0 \end{aligned} \quad (39)$$

Violations of slepton universality occur though several mechanisms. Splittings of the $\tilde{e}, \tilde{\mu}$ and $\tilde{\tau}$ masses depend on the underlying theory of flavor and are expected to be small at least for the first two generations. The most interesting deviation of the above relations occur from sflavor violation in the slepton soft mass squared matrix. For the first two generations, sflavor mixing can introduce dependence on a heavier lepton mass. Sflavor violation in the slepton propagators allows left-right mass squared insertion proportional to m_μ and m_τ for the electron operator, and m_τ for the muon operator. For moderate to large $\tan\beta$ the parametric dependence of the contributions to the electron and muon operators is:

$$d_e \sim \left[(\delta_{12})^{LL}(\delta_{21})^{RR} m_\mu + (\delta_{13})^{LL}(\delta_{31})^{RR} m_\tau\right] \tan\beta \quad (40)$$

$$a_\mu \sim \left[(\delta_{23})^{LL}(\delta_{32})^{RR} m_\tau\right] \tan\beta \quad (41)$$

where

$$(\delta_{ij})^{LL(RR)} \equiv \frac{(\delta m^2_{\tilde{l}_i\tilde{l}_j})^{L,R}}{m^2_{\tilde{l}_{L,R}}} \quad (42)$$

represent insertion of sflavor violating left-left or right-right mass squared mixings in the slepton propagators. The mechanism responsible for introducing slepton flavor violation also affects the left-right higgsino contribution though left-right slepton mixing matrix elements; such that all contributions depend on the specific form of slepton flavor violation in the model, are model dependent, and as such can provide valuable insights into the symmetry of the model. We also include radiative corrections, which can be parametrized using the logarithmic approximation. In the case of LRSUSY, it is significant that the left and right handed slepton mixings are of the same order of magnitude, and thus differ from MSSM where the left-handed slepton mixings dominate [27].

We now proceed to estimate the dominant contributions to the $(g-2)_\mu$, $\mu \to e\gamma$ and electron EDM from general properties of the mixings in order to emphasize the difference between this model and the MSSM. As in MSSM, the chargino-sneutrino mixing dominates the neutralino mixing in the case in which all supersymmetric masses are equal (and of order 100 GeV). This is despite the fact that LRSUSY has 11 neutralinos, and their contribution is overall slightly larger that the corresponding contribution in MSSM: in LRSUSY the contribution of the bino, left-handed wino and left-right-higgsino dominate, when the right-handed scale is above 100 TeV. The slepton flavor contribution coming from the left-right higgsino is very sensitive to the value of Y_{ν_3} and can dominate over a large region of the parameter space. Indeed, if $Y_{\nu_3} \simeq \mathcal{O}(1)$, (which is favored by GUT scenarios with $Y_{\nu_\tau} = Y_t$ at the unification scale) the contribution from the higgsino is one order of magnitude larger than the chargino-sneutrino contribution, dominates the anomalous magnetic moment of the muon, and imposes strict restriction the parameter space $(M_L, m_0, \tan\beta)$ [21]. We will take here $Y_{\nu_3} = 0.3$, in which case the higgsino contribution is of the same order of magnitude as the chargino one. In this case, for $x \simeq 1$ we obtain the values:

$$a_\mu^c \simeq 1.33 \times 10^{-9} \tan\beta \quad (43)$$

$$a_\mu^n + a_\mu^\Phi \simeq 3.9 \times 10^{-9} \tan\beta \quad (44)$$

These estimates have the added advantage that they are fairly constant against variations in values of x for $0.25 \leq x \leq 4$.

Next we compare the anomalous magnetic moment with the flavor-changing dipole operators for $\mu \to e\gamma$. The same three contributions (chargino, neutralino and left-right higgsino) contribute to the radiative muon decay, but now the relative dominance of the higgsino over the chargino contribution is determined by the value of $V_{\tau 1}$. We have two distinct cases:

– (1) If $V_{\tau 1} \leq 10^{-4}$ then the decay $\mu \to e\gamma$ is dominated by the chargino-sneutrino contribution. Taking the mixing of the electron and muon sneutrino to dominate over other sneutrino mixings, we obtain a bound on sneutrino mass splittings:

$$(\delta_{\tilde{\nu}_e\tilde{\nu}_\mu}) \equiv \frac{(\delta m^2_{\tilde{\nu}_\mu\tilde{\nu}_e})}{m^2_{\tilde{\nu}}} < 1.5 \times 10^{-5} \times \left[\frac{B(\mu \to e\gamma)}{1.2 \times 10^{-11}}\right]^{\frac{1}{2}} \left[\frac{4.3 \times 10^{-9}}{a_\mu}\right] \quad (45)$$

for $m_0 \approx m_{1/2} \approx 100$ GeV. (We take $M_1 \simeq 0.5 M_2 \simeq 0.8 m_{1/2}$.) The difference between this relation and the corresponding MSSM one comes from the extra contribution of the left-right higgsino, which is large for a_μ with large stau-selectron mixing, but very small for $\mu \to e\gamma$, due to the small value of the stau-selectron mixing. This results in a more stringent bound on sneutrino mass splitting than the one obtained in MSSM.

– (2) If $V_{\tau 1} > 10^{-4}$ then there is sufficient selectron-stau mixing for the decay $\mu \to e\gamma$ to be dominated by the neutralino-slepton graphs proportional to m_τ. Taking, for the purpose of an estimate $V_{\tau 1} = 10^{-2}$, we obtain a bound on the left and right selectron-smuon mass splittings:

$$(\delta_{\tilde{\mu}_R\tilde{e}_R}) \simeq (\delta_{\tilde{\mu}_L\tilde{e}_L}) \equiv \frac{(\delta m^2_{\tilde{\mu}\tilde{e}})^{L,R}}{m^2_{\tilde{l}_{L,R}}} < 7 \times 10^{-2} \times \left[\frac{B(\mu \to e\gamma)}{1.2 \times 10^{-11}}\right]^{\frac{1}{2}} \left[\frac{4.3 \times 10^{-9}}{a_\mu}\right] \quad (46)$$

This relationship, obtained for $m_0 \approx m_{1/2} \approx 100$ GeV, is less stringent than the one obtained in MSSM. This is due to the difference between the LRSUSY and the MSSM slepton mixing alluded to before. In MSSM, one sets limits on the left-handed slepton mixings, the right-handed mixings being negligible. In LRSUSY the left-handed and right-handed sleptons have LFV masses of the same order. As opposed to SO(10), where the event rate of $\mu \to e\gamma$ is determined by the competition between the neutralino-slepton diagram proportional to m_τ and the chargino-sneutrino diagram [28], in LRSUSY the neutralino diagram proportional to m_τ dominates, for non-negligible $V_{\tau 1}$, due to the specific Higgs-higgsino structure of the model.

Similar results can be obtained for radiative decays of the τ. In the case in which the decays $\tau \to e\gamma$ and $\tau \to \mu\gamma$ are dominated by the chargino-sneutrino contributions:

$$(\delta_{\tilde{\nu}_\tau\tilde{\nu}_\mu}) \equiv \frac{(\delta m^2_{\tilde{\nu}_\tau\tilde{\nu}_\mu})}{m^2_{\tilde{\nu}}} < 2.7 \times 10^{-2} \times \left[\frac{B(\tau \to \mu\gamma)}{1.1 \times 10^{-6}}\right]^{\frac{1}{2}} \left[\frac{4.3 \times 10^{-9}}{a_\mu}\right] \quad (47)$$

and

$$(\delta_{\tilde{\nu}_\tau\tilde{\nu}_e}) \equiv \frac{(\delta m^2_{\tilde{\nu}_\tau\tilde{\nu}_\mu})}{m^2_{\tilde{\nu}}} < 4.2 \times 10^{-2} \times \left[\frac{B(\tau \to e\gamma)}{2.7 \times 10^{-6}}\right]^{\frac{1}{2}} \left[\frac{4.3 \times 10^{-9}}{a_\mu}\right] \quad (48)$$

for $m_0 \approx M_{1/2} \approx 100$ GeV. For neutralino-slepton dominance ($V_{31} > 10^{-4}$) the bounds on stau-smuon and stau-selectron are very weak. This situation corresponds to $|V_{32}| \approx V_{22} \approx V_{33}$, i.e maximal stau-smuon mixing similar to the maximal tau neutrino-muon-neutrino needed to explain the atmospheric neutrino experiments.

Comparing the anomalous magnetic moment of the muon with the electric dipole moment of the electron, we can again look at the dominant terms and derive bounds on either LFV mixing or CP violating angles. In general, the smallness of the EDM is attributed to either the smallness of the CP-violating angle, a heavy mass for the sleptons, or accidental cancellations between the chargino and neutralino contributions [29]. A comment is in order regarding the angle θ_μ. As in MSSM, the fit to the anomalous magnetic moment of the muon is relatively insensitive to the value of CP-violating angles [14]. This justifies neglecting the CP-angles for the purpose of estimating the magnetic moment, but including them in considerations of electric dipole moments. If the electron EDM is dominated by the chargino contribution, its small size compared to the $(g-2)_\mu$ must be due the small phase of $|\mu|$, such that:

$$\tan\theta_\mu < 5.1 \times 10^{-3} \quad (49)$$

for approximately equal supersymmetric masses and $Y_{\nu_3} = 0.3$. This value is approximately 2.5 times as large as in the MSSM under similar circumstances. If the smallness of the electron EDM is due to cancellations between the chargino and neutralino contributions, one has a similar situation as in MSSM: the angles θ_μ and ϕ_1 can be quite large, as long as they are correlated. In that case, the electron EDM can be nearly zero regardless of the value the anomalous magnetic moment of the muon and no new bounds are obtained. If, on the other hand, the angles are such that the electron EDM is dominated by the neutralino-slepton contribution, the electric dipole moment will be proportional to the amount of slepton favor violation as in (40). Assuming that only one off-diagonal slepton mixing domainates, we obtain, again in the limit of equal supersymmetric masses:

$$(\delta_{\tilde{\mu}_R\tilde{e}_R}) \simeq (\delta_{\tilde{\mu}_L\tilde{e}_L}) < 6.2\times 10^{-3}\left[\frac{10^{-1}}{\sin(\theta_\mu+\phi_1)}\right] \times \left[\frac{d_e}{4.3\times 10^{-27} e\ cm}\right]^{\frac{1}{2}}\left[\frac{4.3\times 10^{-9}}{a_\mu}\right] \quad (50)$$

for selectron-smuon dominance, and:

$$(\delta_{\tilde{\tau}_R\tilde{e}_R}) \simeq (\delta_{\tilde{\tau}_L\tilde{e}_L}) < 1.6\times 10^{-3}\left[\frac{10^{-1}}{\sin(\theta_\mu+\phi_1)}\right] \times \left[\frac{d_e}{4.3\times 10^{-27} e\ cm}\right]^{\frac{1}{2}}\left[\frac{4.3\times 10^{-9}}{a_\mu}\right] \quad (51)$$

The first of these bounds appear to be stronger than the bound obtained from $\mu \to e\gamma$; and the second much stronger than the equivalent one obtained from $\tau \to e\gamma$. However, since many more suppositions are involved in these bounds (neutralino-slepton dominance of the EDM and an unknown size of CP-violating angles), in fact these bounds are much less firm than the ones from LFV decays.

All of our analytical results so far are based on one-loop estimates for the dipole moments. However, recent estimates for both the magnetic [30] and electric dipole moments [31] have shown that a sufficiently light pseudoscalar Higgs boson can give significant contributions to dipole moments at two loop level, coming from Barr-Zee

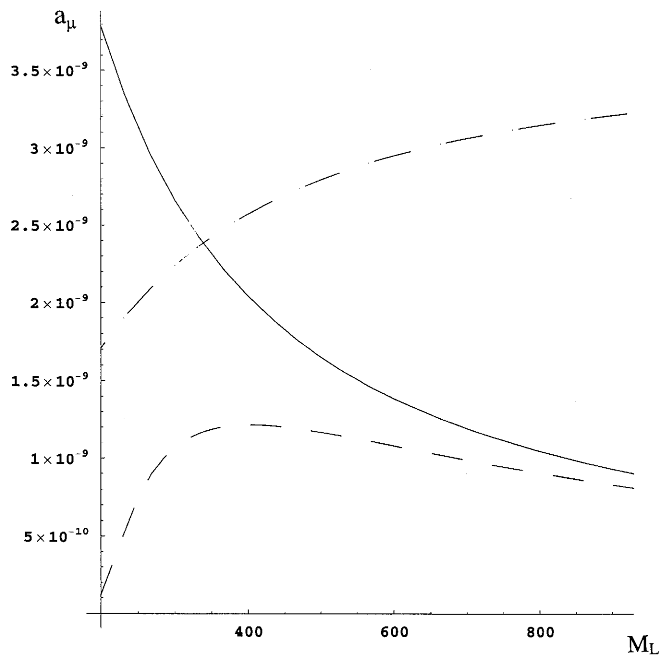

Fig. 2. Chargino, neutralino and left-right higgsino contributions to the anomalous magnetic moment of the muon as a function of the left-handed gaugino mass parameter M_L for fixed universal scalar mass parameter $m_0 = 100$ GeV for $\tan\beta = 5$ and $Y_{\nu_3} = 0.25$. The curves are marked: (solid curve) chargino contribution, (large dashed curve) neutralino contribution, and (dot dashed curve) left-right higgsino contribution. We take $M_R = 100$ TeV in all of our plots

type diagrams. The analysis presented here, in particular the choice of CP-violating angles, does not take such contributions into account. These contributions would depend on parameters from the Higgs and squark sectors. In particular, the electric dipole moment diagrams receive a large contribution from the phase in $Im(\mu A_t)$, whereas in considering one loop diagram we neglected (small) contributions from the imaginary part of the trilinear mixing parameter A_l. The LRSUSY has no new significant two-loop contributions compared to the MSSM. From the general structure of the dipole operator of a Dirac fermion, the contribution to the electron EDM may be related to the anomalous magnetic moment by (38). This relation is independent of which diagrams dominate the dipole operator. If we apply it to the two-loop contributions alone, it can be used to (roughly) bound the phase of the contribution to the dipole operator:

$$\tan|\phi| \leq 2\times 10^{-3} \quad (52)$$

where ϕ is the relevant phase, in this case $Im(\mu A_t)$.

In this section we have shown that the general analytical features of the leptonic sector of the LRSUSY are very different from the ones in MSSM. We now proceed to a more thorough numerical investigation of the relationships between the parameters of the model and the LFV slepton and sneutrino splittings.

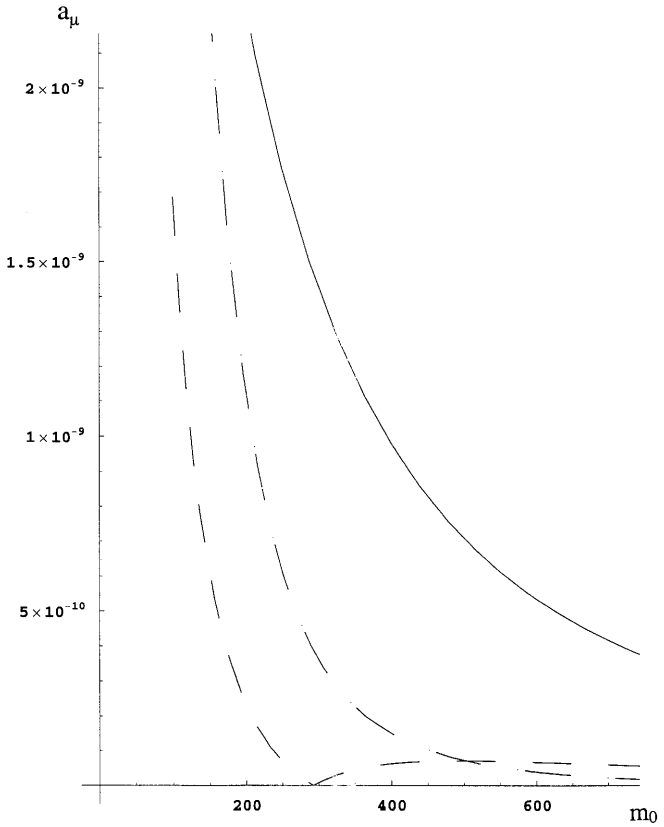

Fig. 3. Chargino, neutralino and left-right higgsino contributions to the anomalous magnetic moment of the muon as a function of the universal scalar mass parameter m_0 for light left-gaugino masses $m_{1/2} = 150$ GeV, $\tan\beta = 5$ and $Y_{\nu_3} = 0.25$. The curves are marked: (solid curve) chargino contribution, (large dashed curve) neutralino contribution, and (dot dashed curve) left-right higgsino contribution

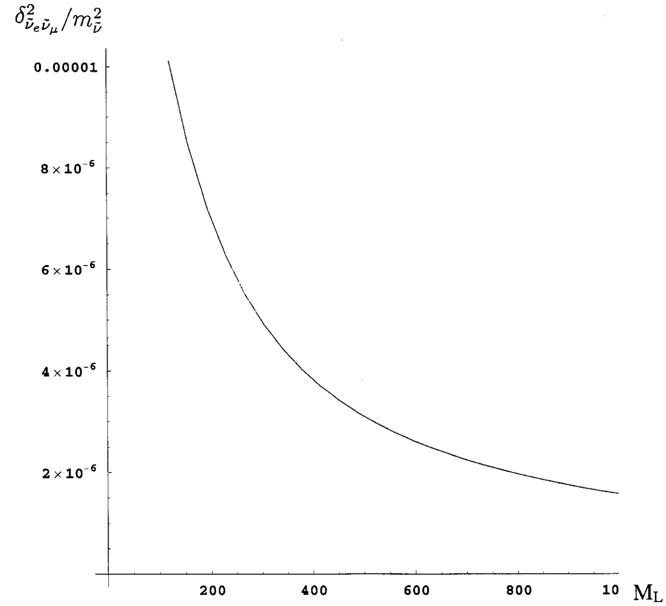

Fig. 4. Fractional mass splitting $\delta m^2_{\tilde{\nu}_e\tilde{\nu}_\mu}/m^2_{\tilde{\nu}}$ as a function of the left-handed gaugino mass for $V_{\tau 1} = 10^{-4}$ (chargino dominance af the anomalous magnetic moment) for $m_0 = 100$ GeV and $\tan\beta = 5$

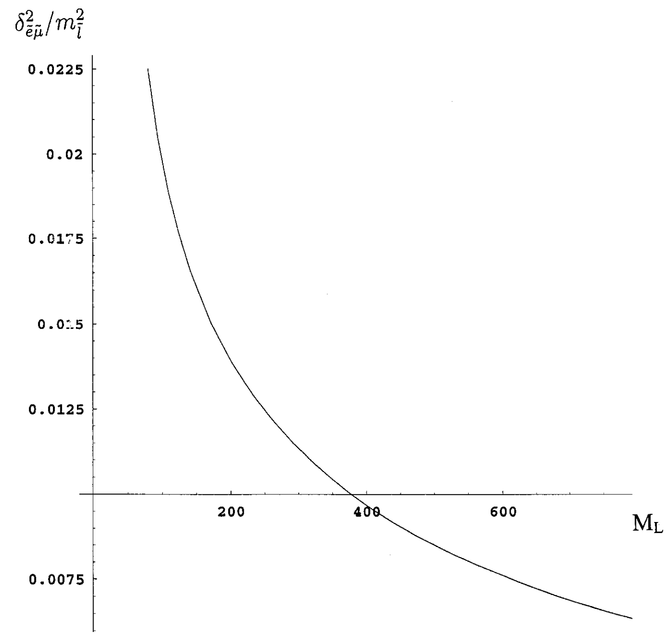

Fig. 5. Fractional mass splitting $\delta m^2_{\tilde{e}\tilde{\mu}}/m^2_{\tilde{l}}$ as a function of the the left-handed gaugino mass for $V_{\tau 1} = 10^{-2}$; $m_0 = 100$ GeV and $\tan\beta = 5$

5 Relationships between dipole operators: The numerical results

The flavor violating decays are sensitive to the universal GUT parameters m_0 (the scalar mass), the trilinear coupling A, the value and the sign of the Higgs mixing parameter μ, the value of $\tan\beta$ the values of the left- and right-handed gaugino masses M_1, M_L and M_R (through $m_{1/2}$). We will assume the sign of μ to be positive, as required by the constrain on the muon anomalous magnetic moment. LFV are not a sensitive probe of the doubly-charged higgsino contribution: although the Yukawa couplings h_{LR} and the mass of the doubly-charged higgsino are not restricted, the contribution from these higgsino is always small. This comes in large part from the fact that all graphs must have chirality flipped on the external lepton leg. We first investigate the dependence of the anomalous magnetic moment of the muon on the gaugino mass parameter. We take $2M_1 \simeq M_L$, as inspired by GUT scenarios, and plot the chargino, neutralino and left-right higgsino contributions as functions of M_L for fixed scalar mass $m_0 = 100$ GeV (Fig. 2). We take in all the figures $M_R = 100$ TeV. The exact value is not essential, as long as it is sufficiently high for the decoupling of the right-handed side of the gaugino sector from the low-energy phenomena. But remnants of left-right symmetry survive and affect the observed slepton/sneutrino spectrum. As expected, the left-right higgsino contribution dominates over most of the spectrum (for $Y_{\nu_3} \simeq 0.25$), with the exception of the case in which the lightest chargino is very light. The neutralino contribution is small compared to

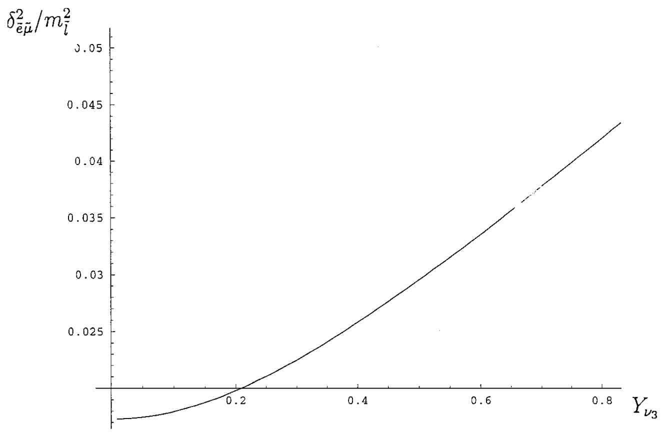

Fig. 6. Fractional mass splitting $\delta m^2_{\tilde{e}\tilde{\mu}}/m^2_{\tilde{l}}$ as a function Y_{ν_3} for $m_{1/2} = 100$ GeV, $m_0 = 100$ GeV and $V_{\tau 1} = 10^{-2}$

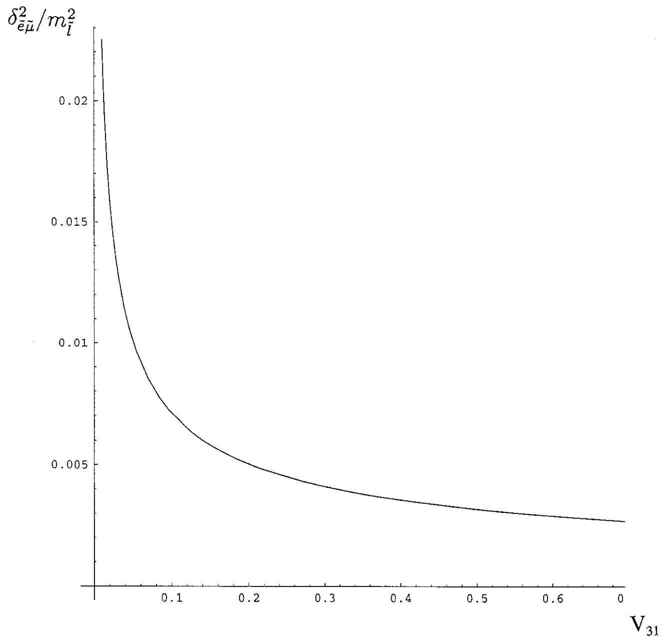

Fig. 7. Fractional mass splitting $\delta m^2_{\tilde{e}\tilde{\mu}}/m^2_{\tilde{l}}$ as a function of $V_{\tau 1}$ for $m_{1/2} = 100$ GeV, $m_0 = 100$ GeV and $Y_{\nu_3} = 0.3$

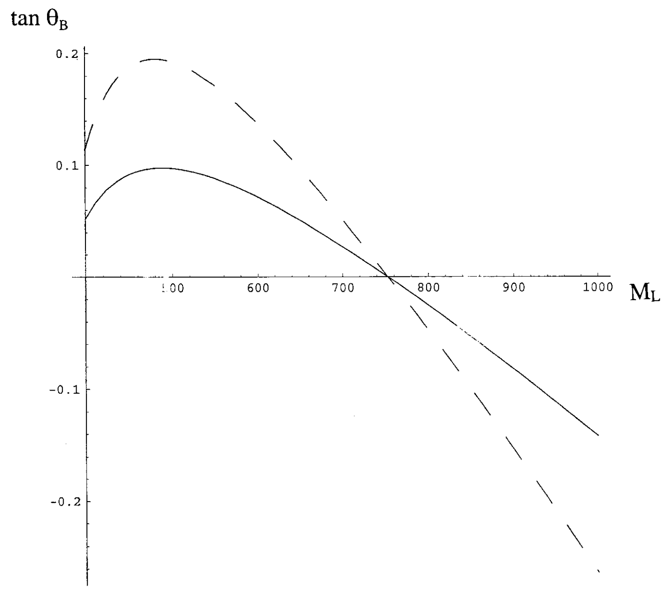

Fig. 8. Variation of the CP-violating angle θ_B as a function of the left-handed gaugino mass parameter M_L for fixed universal scalar mass parameter $m_0 = 100$ GeV for $\tan\beta = 5$ and $Y_{\nu_3} = 0.1$ for two values of the angle ϕ_1: $\phi = 0.5$ (solid curve) and $\phi_1 = 0.7$ (large dashed curve)

the chargino contribution. The same quantities are plotted as a function of the universal scalar mass for very light charginos $m_{1/2} = 150$ GeV (Fig. 3). This case corresponds to the case of chargino-sneutrino dominance (of the left-hand corner of Fig. 2). The left-right higgsino contribution drops off much faster than the chargino contribution with the scalar particle mass, such that in the limit of heavy scalar spectrum, $m_0 \simeq 1$ TeV, the chargino-sneutrino contribution dominates the magnetic moment and LRSUSY contribution resembles more the MSSM one. In Fig. 4, we plot the variation of the lepton-flavor violating electron-muon sneutrino mixing as a function of M_L for $V_{31} = 10^{-4}$ (chargino-sneutrino dominance). The rough expectation, from the previous sections, that the sneutrino masses for the first two generations are close, is satisfied for all gaugino masses and the relative mass splitting drops quickly with the gaugino mass. In the following figures we investigate the dependence of the smuon-selectron mixing on the gaugino mass (Fig. 5), the Yukawa coupling of the tau neutrino (Fig. 6) and $V_{\tau 1}$ (Fig. 7). The slepton mass splitting is very sensitive to the Yukawa coupling of the tau neutrino through the higgsino contribution, which can change from dominant to same order of magnitude as as the rest of the neutralino contribution. As expected, an increase in $V_{\tau 1}$ is related to an increase in the selectron-stau mixing; from unitarity, less mixing is available in the selectron-smuon sector and we expect $\delta_{\tilde{e}\tilde{\mu}}$ to decrease with increasing V_{31} as shown in Fig. 7.

In the next figures we investigate the dependence of the CP-violating angle $\theta_B = -\theta_\mu$ on the left-gaugino mass M_L for two values of the angle ϕ_1 (Fig. 8), on the universal scalar mass m_0 (Fig. 9) and on the angle ϕ_1 (Fig. 10). As in mSUGRA, there exists a strong correlation bewteen the two CP violating angles θ_μ and ϕ_1 in regions in which cancellations between the chargino-sneutrino and the neutralino contributions exist. In these regions large values are allowed for both θ_μ and ϕ_1 as shown in Fig. 8. We plot the allowed values of θ_μ for $\sin\phi_1 = 0.5$ (the solid line) and $\sin\phi_1 = 0.7$ (the dashed line) corresponding to vanishingly small values of the electron EDM. Note that the behavior of these angles is different from mSUGRA and the two graphs intersect, due to interference between various neutralino contributions. The allowed values of θ_μ raise steadily with increased values of the scalar masses; as in MSSM a (vanishingly) small electric dipole moment can be obtained for any angles, if the scalar particles are heavy (Fig. 9). Finally, in Fig. 10 we plot the correlation between the angles θ_μ and ϕ_1: the angles are constrained to be of the same order of magnitude for cancellations to

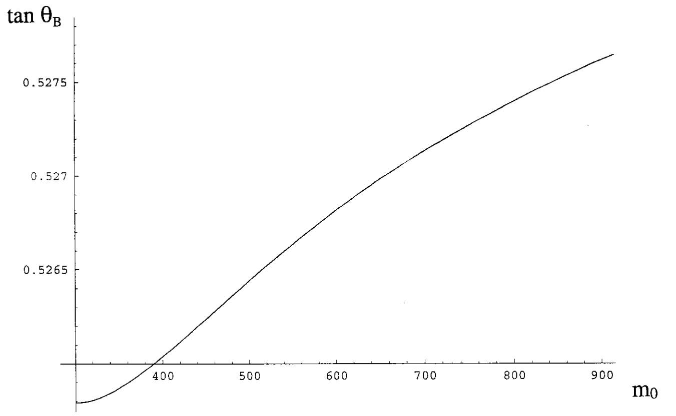

Fig. 9. Variation of the CP-violating angle θ_B with the universal scalar mass parameter m_0 for light left-gaugino masses $m_{1/2} = 100$ GeV, $\tan\beta = 5$ and $Y_{\nu_3} = 0.1$ and $\phi_1 = 0.5$

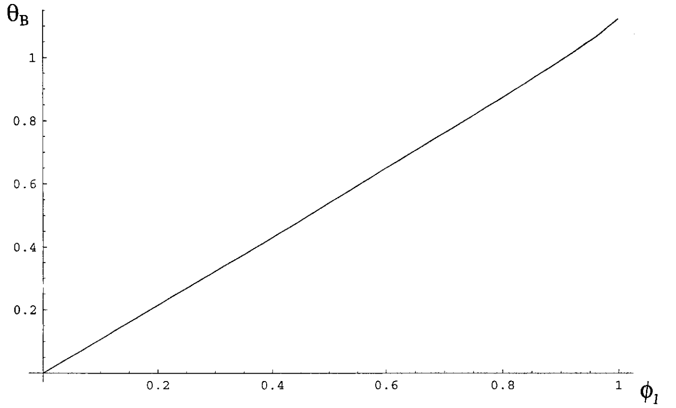

Fig. 10. Variation of the CP-violating angle θ_B with the angle ϕ_1 for $m_{1/2} = 100$ GeV, $m_0 = 100$ GeV, $\tan\beta = 5$ and $Y_{\nu_3} = 0.1$

occur between different contributions, such that the electric dipole moment is always small.

6 Conclusion

The amount of information about leptonic parameters has been growing steadily recently, first with the observation of neutrino oscillations, and then with the observed deviation of the anomalous magnetic moment of the muon from its SM value. The first of these observations indicate a clear departure from the SM (or its minimal supersymmetric extension) and signals the first observed mixing in the leptonic sector. It appears that, with our understanding so far, the leptonic sector behaves very differently from the quark sector. Improved observations in charged lepton phenomenology, such as the new measurement of the muon $g-2$ factor, add another piece to the puzzle. Combined with improvements in restricting further the branching ratios for lepton flavor violating decays (such as $\mu \to e\gamma$ and $\tau \to \mu\gamma$, most notably), these events allow us to formulate a theory of leptonic mixing and decays through the study of leptonic electromagnetic dipole moments. The relative values of various flavor changing and flavor violating decays provide valuable information on mixing in the sneutrino and slepton sectors, as well as on CP violating angles. By analysing these correlations in LRSUSY, a model that incorporates supersymmetry with the seesaw mechanism, we have shown that these correlations are not universal. In LRSUSY models, there are new channels for leptonic flavor violations; and relative mass splittings in either the sneutrino sector or the slepton sector differ from the MSSM. Due to a new contribution from the (neutral) higgsino sector, the sneutrino splitting in LRSUSY is more restricted than in MSSM: $\delta m^2_{\tilde{\nu}_e\tilde{\nu}_\mu}/m^2_{\tilde{\nu}} < 1.5 \times 10^{-5}$. On the other hand, if neutralinos-sleptons dominate the flavor violating decays, these decays pick up equal contributions from both left and right-handed slepton flavor mixing. The relative charged slepton mixing is in this case less restricted than in MSSM: $\delta m^2_{\tilde{e}\tilde{\mu}}/m^2_{\tilde{l}} < 2 \times 10^{-2}$. Weaker boundes on the relative sneutrino mass splittings can be obtained from $\tau \to e\gamma$ and $\tau \to \mu\gamma$: $\delta m^2_{\tilde{\nu}_e\tilde{\nu}_\tau}/m^2_{\tilde{\nu}} < 4.2 \times 10^{-2}$, $\delta m^2_{\tilde{\nu}_\tau\tilde{\nu}_\mu}/m^2_{\tilde{\nu}} < 2.7 \times 10^{-2}$; and in the charged slepton sector, the staus and smuons are allowed to mix maximally. In the CP-violating sector, the correlations between the two angles in the most constrained version of the model are different from mSUGRA, but cancellations between contributions can occur over a large region of the parameter space, such that the two CP-violating angles can both be large while the electron EDM is very small.

The search for slepton/sneutrino oscillations is a powerful tool for probing intergenerational mixings, which in turn depend on assumptions about their origin. As such, the leptonic dipole moments are essential to unraveling the symmetry in the leptonic/sleptonic sector and further expected improvements in experimental searches should shred light on the fundamental gauge symmetry responsible for neutrino masses.

Acknowledgements. This work was funded by NSERC of Canada (SAP0105354).

Appendix

A Chargino and neutralino mixing matrices

The chargino and neutralino masses enter the theory via their mass eigenvalues and mixing matrices. Following [32], we employ the following notation: the U, N matrices rotate the gaugino/Higgsino interaction basis into the neutralino/chargino mass basis. N^0 is the matrix for the neutralinos; U^+ is the matrix for the charginos Ψ_i^+; and U^- is for the charginos Ψ_i^-. $U^{--}_{\Delta_{L,R}}$, $U^{++}_{\Delta_{L,R}}$ are mixing matrices for the doubly charged $\tilde{\Delta}_{L,R}$ and $\tilde{\delta}_{L,R}$ Higgsino.

The electroweak gauginos and Higgsinos are all spin-1/2 weakly interacting charged particles which mix once the symmetry is broken. In the Left-Right Supersymmetric Model, the chargino matrix is a 5x5, non-symmetric, non-Hermitean matrix, M^c, from the Lagrangian (we include here, for the sake of minimizing the number of parameters, only the right-handed triplet Higgsinos):

$$M^c = \begin{pmatrix} M_L & 0 & 0 & \sqrt{2}M_{W_L}\sin\beta & 0 \\ 0 & M_R & 0 & \sqrt{2}M_{W_L}\sin\beta & \frac{(M^2_{W_R}-M^2_{W_L})^{1/2}}{\sqrt{2}} \\ \sqrt{2}M_{W_L}\cos\beta & \sqrt{2}M_{W_L}\cos\beta & 0 & \mu & 0 \\ 0 & 0 & \mu & 0 & 0 \\ 0 & -\frac{(M^2_{W_R}-M^2_{W_L})^{1/2}}{2} & 0 & 0 & M_{LR} \end{pmatrix} \tag{56}$$

$$M^N = \begin{pmatrix} M_V + M_R\tan^2\theta_W & 0 & 2(M_R-M_V) & C_1 & -C_2 & C_4 & -C_4 \\ 0 & M_L & 0 & -C_3 & C_2 & 0 & 0 \\ 2(M_R-M_V) & 0 & M_V+\frac{M_R}{\tan^2\theta_W} & \frac{M_Z\sin\theta_W\cos\beta}{\tan^2\theta_W} & C_2 & -C_5 & C_5 \\ C_1 & -C_3 & \frac{M_Z\cos\theta_W\sin\beta}{\tan^2\theta_W} & 0 & -\mu & 0 & 0 \\ -C_2 & C_2 & C_2 & -\mu & 0 & 0 & 0 \\ C_4 & 0 & -C_5 & 0 & 0 & 0 & M_{LR} \\ -C_4 & 0 & C_5 & 0 & 0 & M_{LR} & 0 \end{pmatrix} \tag{60}$$

$$\mathcal{L}_{ch} = -\frac{1}{2}(\Psi^+ \; \Psi^-)\begin{pmatrix} 0 & M^{cT} \\ M^c & 0 \end{pmatrix}\begin{pmatrix} \Psi^+ \\ \Psi^- \end{pmatrix} + H.C. \tag{53}$$

where

$$\Psi^+ = (-i\lambda^+_L, -i\lambda^+_R, \tilde{\Phi}^+_u, \tilde{\Phi}^+_d, \tilde{\delta}^+_R); \tag{54}$$
$$\Psi^- = (-i\lambda^-_L, -i\lambda^-_R, \tilde{\Phi}^-_u, \tilde{\Phi}^-_d, \tilde{\Delta}^-_R); \tag{55}$$

and (see (56) on top of the page) where: $\tan\beta = \frac{\kappa_d}{\kappa_u}$, M_L, M_R are the gaugino masses in the left- and right-handed sector; and $\chi^-_i = U^-_{ij}\Psi^-_j$, $\chi^+_i = U^+_{ij}\Psi^+_j$, $(i,j=1,...5)$.

As in the MSSM, we need two unitary matrices, $U^{(-)}$ and $U^{(+)}$, to diagonalize M^c:

$$M_D = U^{(-)*}M^cU^{(+)-1} \tag{57}$$

The eigenvalues of M^c can be either positive or negative, whereas we require M_D to have only non-negative entries. We shall use numerical expressions for U^+_{ij} and U^-_{ij} obtained in [32].

The neutralino Lagrangian can be written in matrix form as:

$$\mathcal{L}_n = -\frac{1}{2}(\Psi^0)^T M^n (\Psi^0) + H.C. \tag{58}$$

using the basis (of neutralinos which couple to leptons or sleptons):

$$\Psi^0 = (-i\lambda^0_B\cos\theta_W, -i\lambda^0_L, -i\lambda^0_R\sin\theta_W, \tilde{\Phi}^0_u, \tilde{\Phi}^0_d, \tilde{\Delta}^0_R, \tilde{\delta}^0_R) \tag{59}$$

The neutralino mixing matrix is in general a complex symmetric matrix given by: (see (60) on top of the page) where:

$$C_1 = M_Z\sin\theta_W\cos\beta \tag{61}$$
$$C_2 = M_Z\cos\theta_W\sin\beta \tag{62}$$
$$C_3 = M_Z\cos\theta_W\cos\beta \tag{63}$$
$$C_4 = 2M_{W_R}\cos\theta_W \tag{64}$$
$$C_5 = 2M_{W_R}\sin^2\theta_W/\sqrt{\cos 2\theta_W} \tag{65}$$

Defining the mass eigenstates to be:

$$\chi^0_i = N_{il}\Psi_l, \quad i,l = 1,...7 \tag{66}$$

where N_{il} is a unitary matrix which diagonalizes the neutralino mass matrix:

$$M^N_D = N^*M^NN^{-1} \tag{67}$$

with M^N_D the diagonal neutralino mass matrix. Again, we use the numerical expressions for N_{ij} obtained in [32].

B The complete contributions to the anomalous magnetic moment of the muon with slepton and sneutrino mixing

The contributions to the anomalous magnetic moment of the muon in the Left-Right Supersymmetric Model are presented in the diagrams of Fig. 1. We write the contributions as:

$$a_{L\mu} = a^{c,0}_{L\mu} + a^{n,0}_{L\mu} + a^c_{L\mu} + a^{n,1}_{L\mu} + a^{n,2}_{L\mu} + a^{\phi}_{L\mu} + a^{\Delta_L}_{L\mu} \tag{68}$$
$$a_{R\mu} = a^{n,0}_{R\mu} + a^{n,1}_{R\mu} + a^{n,2}_{R\mu} + a^{\phi}_{R\mu} + a^{\Delta_R}_{R\mu} \tag{69}$$

Here A_L represents the left-handed contribution, and A_R the contribution from the right-handed sector, as given below.

We present first the individual contributions for graphs with chirality flip on the external fermion line.
For charginos, left-handed fermions, with an external chirality flip (Fig. 1a):

$$a^{c,0}_{L\mu} = -\frac{g^2m_\mu}{16\pi^2}(U_{W_Lk})^2 \times\left[|K_{\mu1}|^2\frac{f(x_{ke})}{m^2_{\tilde{\nu}_e}} + |K_{\mu2}|^2\frac{f(x_{k\mu})}{m^2_{\tilde{\nu}_\mu}} + |K_{\mu3}|^2\frac{f(x_{k\tau})}{m^2_{\tilde{\nu}_\tau}}\right] \tag{70}$$

For neutralinos, left-handed fermions, with an external chirality flip (Fig. 1b):

$$a^{n,0}_{L\mu} = \frac{g^2 m_\mu}{16\pi^2}\left(N^0_{W_L k} + \tan\theta^2_W N^0_{Bk}\right)^2 \times \left[|V^L_{\mu1}|^2 2\frac{g(y_{ke_L})}{m^2_{\tilde{e}_L}} + |V^L_{\mu2}|^2\frac{g(y_{k\mu_L})}{m^2_{\tilde{\mu}_L}} + |V^L_{\mu3}|^2\frac{g(y_{k\tau_L})}{m^2_{\tilde{\tau}_L}}\right] \tag{71}$$

For neutralinos, right-handed fermions, with an external chirality flip (Fig. 1c):

$$a^{n,0}_{R\mu} = \frac{g^2 m_\mu}{16\pi^2}\left(N^0_{W_R k} - 2\tan\theta^2_W N^0_{Bk}\right)^2 \times \left[|V^R_{\mu1}|^2\frac{g(y_{ke_R})}{m^2_{\tilde{e}_R}} + |V^R_{\mu2}|^2\frac{g(y_{k\mu_R})}{m^2_{\tilde{\mu}_R}} + |V^R_{\mu3}|^2\frac{g(y_{k\tau_R})}{m^2_{\tilde{\tau}_R}}\right] \tag{72}$$

For doubly-charged Higgsinos, left-handed fermions, with an external chirality flip (Fig. 1d):

$$a^{\Delta_L}_{L\mu} = -\frac{h_{LR\tau}h_{LR\mu}}{16\pi^2}\left(U_{\Delta^{++}_L}\right)^2\left[|V^L_{\mu1}|^2\frac{(f+2g)(y_{ke_L})}{m^2_{\tilde{e}_R}} + |V^L_{\mu2}|^2\frac{(f+2g)(y_{k\mu_L})}{m^2_{\tilde{\mu}_L}} + |V^R_{\mu3}|^2\frac{(f+2g)(y_{k\tau_L})}{m^2_{\tilde{\tau}_L}}\right] \tag{73}$$

For doubly-charged Higgsinos, right-handed fermions, with an external chirality flip (Fig. 1e):

$$a^{\Delta_R}_{R\mu} = -\frac{h_{LR\tau}h_{LR\mu}}{16\pi^2}\left(U_{\Delta^{++}_R}\right)^2\left[|V^R_{\mu1}|^2\frac{(f+2g)(y_{ke_R})}{m^2_{\tilde{e}_R}} + |V^R_{\mu2}|^2\frac{(f+2g)(y_{k\mu_R})}{m^2_{\tilde{\mu}_R}} + |V^R_{\mu3}|^2\frac{(f+2g)(y_{k\tau_R})}{m^2_{\tilde{\tau}_R}}\right] \tag{74}$$

Next, we present the expressions for the graphs where chirality is flipped at the vertex. For most of the parameter space, these graphs are dominant over the graphs where the chiralty is flipped externally, unless they involve heavy sneutrinos.
The chargino-neutrino contribution with vertex chirality flip is (Fig. 1f):

$$a^c_{L\mu} = \frac{m_\mu}{16\pi^2}M_{\chi^\pm} g Y_\mu Re[U^{-*}_{k1}U^{+*}_{k1}]\left[|K_{\mu1}|^2\frac{h(x_{ke})}{m^2_{\tilde{\nu}_e}} + |K_{\mu2}|^2\frac{h(x_{k\mu})}{m^2_{\tilde{\nu}_\mu}} + |K_{\mu3}|^2\frac{h(x_{k\tau})}{m^2_{\tilde{\nu}_\tau}}\right] \tag{75}$$

The neutralino-left slepton contribution with vertex chirality flip (Fig. 1g):

$$a^{n,1}_{L\mu} = \frac{m_\mu}{16\pi^2}M_{\chi^0}\sqrt{2}gY_\mu\left(N^0_{W_L k} + \tan\theta^2_W N^0_{Bk}\right)N^0_{H_k} \times \left[|V^L_{\mu1}|^2 2\frac{j(y_{ke_L})}{m^2_{\tilde{e}_L}} + |V^L_{\mu2}|^2\frac{j(y_{k\mu_L})}{m^2_{\tilde{\mu}_L}} + |V^L_{\mu3}|^2\frac{j(y_{k\tau_L})}{m^2_{\tilde{\tau}_L}}\right] \tag{76}$$

The neutralino-right slepton contribution with vertex chirality flip is (Fig. 1h):

$$a^{n,1}_{R\mu} = \frac{m_\mu}{16\sqrt{2}\pi^2}M_{\chi^0}gY_\mu\left(N^0_{W_R k} - 2\tan\theta^2_W N^0_{Bk}\right)N^0_{H_k} \left[|V^R_{\mu1}|^2\frac{j(y_{ke_R})}{m^2_{\tilde{e}_R}} + |V^R_{\mu2}|^2\frac{j(y_{k\mu_R})}{m^2_{\tilde{\mu}_R}} + |V^R_{\mu3}|^2\frac{j(y_{k\tau_R})}{m^2_{\tilde{\tau}_R}}\right] \tag{77}$$

Finally we give the expressions for the case in which chirality is flipped on the internal slepton line.
The neutralino-left slepton contribution with internal line chirality flip is (Fig. 1i):

$$a^{n,2}_{L\mu} = \frac{m_\mu}{16\pi^2}M_{\chi^0}g^2\left(N^0_{W_R k} - 2\tan\theta_W N^0_{Bk}\right) \times \left(N^0_{W_L k} + \tan\theta^2_W N^0_{Bk}\right)\left[\frac{\mathcal{A}_e}{m^2_{\tilde{e}_R}}V^L_{\mu1}V^{R*}_{\mu1}\frac{j(y_{ke_L})}{m^2_{\tilde{e}_L}} + \frac{\mathcal{A}_\mu}{m^2_{\tilde{\mu}_R}}V^L_{\mu2}V^{R*}_{\mu2}\frac{j(y_{k\mu_L})}{m^2_{\tilde{\mu}_L}} + \frac{\mathcal{A}_\tau}{m^2_{\tilde{\tau}_R}}V^L_{\mu3}V^{R*}_{\mu3}\frac{j(y_{k\tau_L})}{m^2_{\tilde{\tau}_L}}\right] \tag{78}$$

The neutralino-right slepton contribution with internal line chirality flip is (Fig. 1j):

$$a^{n,2}_{R\mu} = \frac{m_\mu}{16\pi^2}M_{\chi^0}g^2\left(N^0_{W_L k} + \tan\theta^2_W N^0_{Bk}\right) \times \left(N^0_{W_R k} - 2\tan\theta_W N^0_{Bk}\right)\left[\frac{\mathcal{A}_e}{m^2_{\tilde{e}_L}}V^R_{\mu1}V^{L*}_{\mu1}\frac{j(y_{ke_R})}{m^2_{\tilde{e}_R}} + \frac{\mathcal{A}_\mu}{m^2_{\tilde{\mu}_L}}V^R_{\mu2}V^{L*}_{\mu2}\frac{j(y_{k\mu_R})}{m^2_{\tilde{\mu}_R}} + \frac{\mathcal{A}_\tau}{m^2_{\tilde{e}_L}}V^R_{\mu3}V^{L*}_{\mu3}\frac{j(y_{k\tau_R})}{m^2_{\tilde{\tau}_R}}\right] \tag{79}$$

The left-right Higgsino-slepton contribution with internal chirality flip is (Fig. 1k):

$$a^\phi_\mu = \frac{2Y^2_{\mu\tau}}{(4\pi)^2}m_\mu m_\tau Re[\mu\tan\beta - A^*_\tau]\frac{M_{\tilde{\Phi}^0_{2u}}}{m^2_{\tilde{\tau}_R} - m^2_{\tilde{\tau}_L}} \times \left[\frac{j(y_{\tau R})}{m^2_{\tilde{\tau}_R}} - \frac{j(y_{\tau_L})}{m^2_{\tilde{\tau}_L}}\right] \tag{80}$$

We have taken into account that the right-handed sneutrinos decouple and therefore there are no chargino-right sneutrino contributions to the anomalous magnetic moment of the muon. Note also that chirality cannot be flipped internally in the graphs with doubly-charged Higgsinos. The dipole loop functions are:

$$f(x) = \frac{1}{12(1-x)^4}\left(x^3 - 6x^2 + 3x + 2 + 6x\log x\right) \tag{81}$$

$$g(x) = \frac{1}{12(1-x)^4}\left(2x^3 + 3x^2 - 6x + 1 - 6x^2\log x\right) \tag{82}$$

$$h(x) = -\frac{1}{2(1-x)^3}\left(x^2 - 4x + 3 + 2\log x\right) \tag{83}$$

$$j(x) = \frac{1}{2(1-x)^3}\left(-x^2 + 1 + 2x\log x\right) \tag{84}$$

The generalization to $l_i \to l_j\gamma$ and the electric dipole moment of the electron proceeds as follows: $a_{\mu e\gamma} \sim -\frac{e}{2m_\mu} a_{\mu e}$, where $a_{\mu e}$ stands for a_μ with the second mixing matrix element involving muon sneutrino/smuon mixing $K(V^{L,R})_{\mu i}$ replaced by the corresponding matrix element for electron sneutrino/selectron mixing $K(V^{L,R})_{ei}$. For the electric dipole of the electron, $d_e \sim -\frac{em_e}{2m_\mu^2} a_e \tan\theta$, where a_e is the amplitude with both matrix elements $K(V^{L,R})_{\mu i}$ replaced by the corresponding electron sneutrino/selectron ones $K(V^{L,R})_{ei}$ and $\tan\theta$ is the corresponding CP-angle (θ_μ or $\theta_\mu+\phi_1$). An example is provided in the text. Explicit formulas for the amplitude for $\mu \to e\gamma$ and the electron EDM have appeared before [23,24].

References

1. H. N. Brown et al., BNL E821 Collaboration, hep-ex/0102017
2. Y. Fukuda et.al. Phys. Lett. B **433**, 9 (1998); Phys. Lett. B **436**, 33 (1998); Phys. Rev. Lett. **81**, 1562 (1998)
3. L. Everett, G. L. Kane, S. Rigolin, L. Wang, hep-ph/0102145; J.L. Feng, K.T.Matchev, hep-ph/0102146; E.A. Baltz, P. Gondolo, hep-ph/0102147; U. Chattopadhayay, P. Nath, hep-ph/0102157; T. Kobayashi, H. Terao, hep-ph/0103028; S. Komine, T. Moroi, M. Yamaguchi, hep-ph/0102204; ibd. hep-ph/0103182; R. Arnowitt, B. Dutta, B.Hu, Y. Santoso, hep-ph/0102344; K. Choi, K. Hwang, S.K. Kang, K.Y. Lee, hep-ph/010348; S. P. Martin, J.D. Wells, hep-ph/0103067; K. Cheung, C. H. Chou, O.C.W.Kong, hep-ph/0103183; S. Baek, P.Ko, H.S. Lee, hep-ph/0103218; D.F. Carvalho, J. Ellis, M.E. Gómez, S. Lola, hep-ph/0103256; H. Baer, C. Bal'azs, J. ferrandis, X. Tata, hep-ph/0103280; K. Enqvist, E. Gabrielli K. Huitu, hep-ph/0104174
4. R. Mohapatra, G. Senjanović, Phys. Rev. Lett. **44**, 912 (1980)
5. P. Langacker, M. Luo, Phys. Rev. D **44**, 817 (1991); G. Ross, R.G. Roberts Nucl. Phys. B **377**, 571 (1992); J.Ellis, S.Kelley, D.V. Nanopoulos, Phys. Lett. B **260**, 131 (1995)
6. M. Cvetč, J. Pati, Phys. Lett. B **135**, 57 (1984); R. N. Mohapatra, A. Rašin, Phys. Rev. D **54**, 5835 (1996); R. Kuchimanchi, Phys. Rev. Lett. **79**, 3486 (1996); R.N. Mohapatra, A. Rašin, G. Senjanović, Phys. Rev. Lett. **79**, 4744 (1997); C.S. Aulakh, K. Benakli, G. Senjanović, Phys. Rev. Lett. **79**, 2188 (1997); C.Aulakh, A. Melfo, G. Senjanović, Phys. Rev. D **57** (1998) 4174
7. J.C. Pati, A. Salam, Phys. Rev. D **10**, 275 (1974); R.N. Mohapatra, J.C. Pati, Phys. Rev. D **11**, 566 (1975), 2558; G. Senjanovič, R.N. Mohapatra, Phys. Rev. D **12**, 1502 (1975); R.N. Mohapatra, R.E. Marshak, Phys. Lett. B **91**, 222 (1980)
8. R. Francis, M. Frank, C.S. Kalman, Phys. Rev. D **43**, 2369 (1991)
9. K. Huitu, J. Maalampi, Phys. Lett. B **344**, 217 (1995); K. Huitu, J. Maalampi, M. Raidal, Phys. Lett. B **328**, 60 (1994); K. Huitu, J. Maalampi, M. Raidal, Nucl. Phys. B **420**, 449 (1994)
10. K.S. Babu, S.M. Barr, Phys. Rev. D **48**, 5354 (1993); K.S. Babu, S.M. Barr, Phys. Rev. D **50**, 3529 (1994); M. Frank, H. Hamidian, K. Puolamäki, Phys. Lett. B **456**, 179 (1999); M. Frank, H. Hamidian, K. Puolamäki, Phys. Rev. D **60**, 095011 (1999); For a review and further references see e.g. R.N. Mohapatra, hep-ph/9801235
11. G. Aldazabal, L. Ibanez, F. Quevedo, hep-ph/0005033
12. M. Frank, H. Saif, Z. Phys. C **67**, 32 (1995); Z. Phys. C **69**, 673 (1996); Mod. Phys. Lett. A **11**, 2443 (1996); J. Phys. G **22**, 1653 (1996)
13. M. Graesser, Scott Thomas, hep-ph/0104254; Z. Chacko, Graham D. Kribs, hep-ph/0104317
14. R. Arnowitt, B. Dutta, Y. Santoso, hep-ph/0106089
15. Z. Chacko, R. N. Mohapatra, hep-ph/9712359; B. Dutta, R.N Mohapatra, hep-ph/9804277
16. R. Kuchimanchi, R.N.Mohapatra Phys. Rev. D **48**, 4352 (1993)
17. A. Pilaftsis, Phys. Rev. D **52**, 459 (1995); A. Pliaftsis, J. Bernabéu, Phys. Lett. B **351**, 235 (1995)
18. G. Couture, M. Frank, H. König, M. Pospelov, Eur. Phys. J. C **7**, 135 (1999)
19. M.E. Gómez, G.K. Leontaris, S. Lola, J.D. Vergados, Phys. Rev. D **59**, 116009 (1999)
20. J. Ellis, M.E. Gómez, G.K. Leontaris, S. Lola, D.V. Nanopoulos, Eur. Phys. J. C **14**, 319 (2000)
21. M. Frank, Mod. Phys. Lett. A **16**, 795 (2001)
22. M. L. Brooks et. al [MEGA Collaboration], Phys. Rev. Lett. **83**, 1521 (1999)
23. M. Frank, Phys. Rev. D **59**, 013003 (1999); M. Frank, Eur. Phys. J. C **17**, 501 (2000)
24. M. Frank Phys. Rev. D **59**, 055006 (1999)
25. Particle Data Group, Eur. Phys. J. C **15**, 1 (2000)
26. L. M. Barkov et al., a research proposal to PSI, "Search for $\mu^+ \to e^+\gamma$ down to 10^{-14} branching ratio" (1999)
27. J. Hisano, T. Moroi, K. Tobe M. Yamaguchi, T. Yanagida, Phys. Lett. B **358**, 579 (1998); J. Hisano, D. Nomura, T. Yanagida, Phys. Lett. B **437**, 351 (1998). J. Hisano, D. Nomura Phys. Rev. D **59**, 11605 (1999)
28. Y. Okada, K. Okumura, Y. Shimizu, Phys. Rev. D **61**, 094001 (2000)
29. T. Ibrahim, P. Nath, Phys. Lett. B **418**, 98 (1998); Phys. Rev. D **57**, 478 (1998); Erratum ibid. Phys. Rev. D **58**, 019901 (1998); Phys. Rev. D **58**, 111301 (1998); T. Falk, K. Olive, Phys. Lett. B **439**, 71 (1998); M. Brhlik, G. J. Good, G. L. Kane, Phys. Rev. D **59**, 115004 (1999); A. Bartl, T. Gajdosik, W. Porod, P. Stockinger, H. Stremnitzer, Phys. Rev. D **60**, 073003 (1999); S. Pokoski, J. Rosiek, C. A. Savoy, Nucl. Phys. B **570**, 81 (2000)
30. D. Chang, W-F. Chang, C-H. Chou, W-Y. Keung, Phys. Rev. D **63**, 091301(R) (2001)
31. D. Chang, W-Y. Keung, A. Pilafsis, Phys. Rev. Lett. **82**, 900 (1999); Erratum ibid. **83**, 3972 (1999); A. Pilaftsis, Phys. Lett. B **471**, 174 (1999); D. Chang, W-F. Chang, W-Y. Keung, Phys. Lett. B **478**, 239 (1999)
32. M. Frank, C. S. Kalman, H. Saif Z. Phys. C **59**, 655 (1993)